FOYDALANILGAN ADABIYOTLAR

1. "O'QUVCHILARNI MATEMATIK OLIMPIADALARGA TAYYORLASH" M.A. Mirzahmedov.

2. "МАТЕМАТИКА В ШКОЛЕ" va "КВАНТ" (Rossiya nashrlari) jurnallarining turli yillardagi sonlari.

1. Tenglama butun sonlarda nechta yechimga ega:

$$\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}=\frac{1}{\sqrt{1000}}$$

2. Tenglamani yeching: $x^{\log_2 9}=x^2\cdot 3^{\log_2 x}-x^{\log_2 3}$

3. 19^{8^7} ning oxirgi uchta raqamini toping.

4. Tenglamalar sistemasini yeching:

$$\begin{cases} x^4+y^4+z^4=1 \\ x^2+y^2+2z^2=\sqrt{7} \end{cases}$$

5. $0<x<\frac{\pi}{2}$ uchun $2^{\sin x}+2^{\operatorname{tg} x}\geq 2^{x+1}$ tengsizlikni isbotlang.

6. Tenglama butun sonlarda nechta yechimga ega?

$$x^2-3y^2=1$$

7. Tenglama butun sonlarda nechta yechimga ega?

$$x^2+y^3=z^4$$

8. Grafigini yasang: $\frac{1}{x^2+1}+\frac{1}{y^2+1}=\frac{2}{xy+1}$

9. Agar $x+y=z+t$ bo'lsa, $(x,y,z,t \in Z)$ $x^2+y^2+z^2+t^2$ ifoda 3 ta butun sonning kvadratlari yig'indisiga teng bo'lishini isbotlang.

10. Tenglamani yeching: $\sqrt{x^3+24}=3x+8+\sqrt{x^3+12}$

11. Ifodani soddalashtiring. $\frac{\left(1^4+\frac{1}{4}\right)\left(3^4+\frac{1}{4}\right)\ldots\left(19^4+\frac{1}{4}\right)}{\left(2^4+\frac{1}{4}\right)\left(4^4+\frac{1}{4}\right)\ldots\left(20^4+\frac{1}{4}\right)}$

12. Tengsizlikni isbotlang: $\frac{x}{y}+\sqrt{\frac{y}{z}}+\sqrt[3]{\frac{z}{x}}>\frac{3}{2}$

13. Agar $xy+\sqrt{(1+x^2)(1+y^2)}=a$ bo'lsa, $x\sqrt{1+y^2}+y\sqrt{1+x^2}$ ni toping.

14. $n\geq3$ (n-natural son) uchun quyidagi tengsizlik bajarilishini isbotlang:

$$\sqrt[3]{n}>\sqrt[4]{n+1}$$

15. Agar *a*,*b*,*c*-uchburchak tomonlar va A,B,C-ular qarshisidagi burchaklar bo'lsa va $ab\cos C+bc\cos A+ac\cos B=c^2$ tenglik bajarilsa, bu uchburchak to'g'ri burchakli ekanini isbotlang.

16. ABC uchburchakda $\text{tg}A=\frac{1}{2}\text{tg}B=\frac{1}{3}\text{tg}C$ munosabat o'rinli bo'lsa, *a:b:c* ni toping.

17. α^β soni ratsional bo'ladigan irratsional α va β lar mavjudmi?

18. Agar $x^5+y^5=x-y$ va $x\geq y>0$ bo'lsa, $x^4+y^4<1$ ni isbotlang.

19. Tenglamani yeching: [sinx]{sinx}=sinx
([]-sonning butun qismi, {}-kasr qismi)

20. Tenglamani yeching: $\frac{1}{\{x\}}+\frac{1}{[x]}=\frac{1}{x}$

21. Tenglamani yeching: $2\log_6(\sqrt{x}+\sqrt[4]{x})=\log_4 x$

22. Shunday *a*,*b*,*c* butun sonlarni topingki,
$\sqrt{9-8\sin 50^\circ}=a+b\sin c^\circ$ tenglik bajarilsin.

23. Tengsizlik to'g'rimi?

$$\sqrt{40\sqrt{38\sqrt{36\sqrt{34...\sqrt{6\sqrt{2}}}}}}<105$$

24. Tenglamani yeching: $x^4-4x^3-1=0$

25. Tenglamani yeching:
$a\sqrt{x^2-b^2-c^2}+b\sqrt{x^2-a^2-c^2}+c\sqrt{x^2-a^2-b^2}=a^2+b^2+c^2$

26. 2^n+n^2 ifoda 100 ga bo'linadigan n-natural sonlar cheklimi yoki cheksizmi?

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27. Istalgan natural n da tengsizlik o'rinli bo'lishini isbotlang:

$$\sqrt{2\sqrt{3\sqrt{4\ldots\sqrt{n}}}} < 3$$

28. $\{x_n\}$ ketma-ketlik $x_{n+1}=x_n^2-2x_n+2$ shart bilan berilgan. $x_{10}=x_1$ bo'lishi uchun x_1 qanday bo'lishi kerak?

29. Barcha shunday *a* va *b* tub sonlarni topingki, $a^{a+1}+b^{b+1}$ ham tub son bo'lsin.

30. Tenglamani yeching: $\sqrt[3]{2-x}+\sqrt{x-1}=1$

31. Tenglamalar sistemasini yeching:

$$\begin{cases} \dfrac{x}{\sqrt{y}}+\dfrac{y}{\sqrt{x}}=xy \\ x^\alpha+y^\alpha=8(xy)^{\frac{\alpha-3}{2}} \end{cases}, \quad (\alpha \in R)$$

32. Tengsizlikni isbotlang:

$$\frac{1}{a^3+b^3+abc}+\frac{1}{b^3+c^3+abc}+\frac{1}{c^3+a^3+abc}\le\frac{1}{abc}, \quad (a,b,c>0)$$

33. Tenglamani natural sonlarda yeching:

$$(x^2+y^2)(z^2+t^2)=4(xz+yt)^2$$

34. Tenglamani yeching: $x+\sqrt{x+\frac{1}{2}+\sqrt{x+\frac{1}{4}}}=a$

35. Sonning butun qismini toping:

$$\sqrt{6+\sqrt{6+\sqrt{6+\ldots+\sqrt{6}}}}+\sqrt[3]{6+\sqrt[3]{6+\sqrt[3]{6+\ldots+\sqrt[3]{6}}}}$$

36. 2^n+4^k (n,k Є N) ko'rinishdagi nechta aniq kvadrat son bor?

37. Quyidagi tengsizlik to'g'rimi?

$$\sqrt[2006]{2006!} > \sqrt[2007]{2007!}$$

38. Tenglamani yeching: $(4^x+2)(2-x)=6$

39. Tengsizlikni isbotlang: $\sin^{2k}\alpha+\cos^{2k}\alpha\le2(\sin^{2k+2}\alpha+\cos^{2k+2}\alpha)$, $(\alpha \in R)$

40. Qanday natural son x,y,z larda tengsizlik bajariladi: $xyz<xy+yz+xz$?

41. Tenglamani yeching:

$$\left(\frac{1}{1\cdot 101}+\frac{1}{2\cdot 102}+...+\frac{1}{10\cdot 110}\right)x=\frac{1}{1\cdot 11}+\frac{1}{2\cdot 12}+...+\frac{1}{100\cdot 110}$$

42. Tenglamani yeching: $\frac{x}{x^2+7x+a}=\frac{x^2+8x+a}{x^2+6x+a}$ (a Є R)

43. Tenglamalar sistemasini yeching:

$$\begin{cases} x^3=y^2 \\ x+y+\sqrt[5]{xy}=819 \end{cases}$$

44. 9 ta bola 220 ta qo'ziqorin terdi. Bunda ixtiyoriy ikkitasi turli miqdorda qo'ziqorin terishdi. Shunday 5 ta boladan iborat guruh topilishini isbotlangki, ularning tergan qo'ziqorinlari yig'indisi 110 tadan oshmasin.

45. Tenglamani yeching: $2(2\cos 4x+1)\cos x=1$

46. Ixtiyoriy uchburchak uchun quyidagi tengsizlik bajarilishini isbotlang: $(m_a^2+m_b^2+m_c^2)(h_a^2+h_b^2+h_c^2)\geq 27S^2$. (Bu yrda *a,b,c*- uchburchak tomonlari, *m*-mediana, *h*-balandlik, *S*-yuza)

47. Agar $f(x)=\frac{x\sqrt{3}-1}{x+\sqrt{3}}$ bo'lsa, $g(x)=\underbrace{f(f...(f}_{2007}(x))...)$ ni toping.

48. Agar $\sqrt{1+x}+\sqrt{1+y}=2\sqrt{1+a}$ bo'lsa, $x+y\geq 2a$ ni isbotlang.

49. $\{a_n\}$ ketma-ketlik $a_1=1$, $a_2=1$, $a_3=2$, $a_{n+3}=\frac{a_{n+1}a_{n+2}+5}{a_n}$, ($n\geq 1$) shart bilan berilgan. Bu ketm-ketlikning barcha hadlari butun son bo'lishini isbotlang.

50. Ixtiyoriy uchburchak uchun $h_a\leq\sqrt{bc}\cos\frac{\alpha}{2}$ tengsizlik bajarilishini isbotlang. (Bunda α- *a* tomon qarshisidagi burchak, h_a-*a* tomonga tushirilgan balandlik)

51. Tengsizlikni isbotlang: $\frac{tg^5\alpha+tg^5\beta+tg^5\gamma}{tg\alpha+tg\beta+tg\gamma}\geq 9$ (α,β,γ-o'tkir burchakli uchburchak burchaklari

171. J: $x_1=2$, $x_2=3$

172. $\frac{19n+91}{n+17}=\frac{19(n+17)-232}{n+17}=19-\frac{232}{n+17}$.
232 ning bo'luvchilari esa: 1,2,4,8,29,58,116,232; $n+17\geq 18$, $n\geq 1$.
Tekshirishlar shuni ko'rsatadiki, n=12,41,99,215. J: 4 ta

173. 1) $m_a+m_b>m_c$, $m_b+m_c>m_a$, $m_a+m_c>m_b$ da yeching.

174. Mustaqil yechishga urinib ko'ring.

175. Tengsizlikdagi qavslarni ochsak: $x-xy+y-yz+z-zx<1$ ga keladi.
$xyz<0$, va $(x-1)(y-1)(z-1)<0$ tengsizliklarni qo'shsak:
$xyz+x+y+z-xy-yz-xz-1<0$, $xyz<0$ bo'lgani uchun uni tashlab yuborsak:
$x+y+z-xy-yz-xz-1<0$, ↔ $x(1-y)+y(1-z)+z(1-x)<1$

176. ∠ABO=90º-φ , ∠AOB=90º, ∠OAC=α-φ. Mustaqil davom ettiring.

177. $x+\frac{1}{x}=n$ tenglamaning yechimi, $x=\frac{n+\sqrt{n^2-4}}{2}$. $x^m+\left(\frac{1}{x}\right)^m=k_m$ bo'lsin. U holda $k_{m+1}=k_m\left(x+\frac{1}{x}\right)-k_{m-1}=nk_m-k_{m-1}$, $\left(x+\frac{1}{x}\right)$-butun son, shuning uchun $k=x^m+\left(\frac{1}{x}\right)^m$ ham butun son, ixtiyoriy m da $x^m=\frac{k+\sqrt{k^2-4}}{2}$ o'rinli.

178. Ko'rinib turibdiki, n>3. Agar n=4 bo'lsa, 10000:2006=4,9850...
Endi vergulni o'ng tomonga raqam 5 dan kichik bo'lguncha suramiz. U holda 3 xona suriladi. 4+3=7, demak, n=7. J: n=7 da

179. O'tkir burchakli uchburchak uchun $tgA+tgB+tgC=tgAtgBtgC$ o'rinli.
Koshi tengsizligiga ko'ra, $tgA+tgB+tgC\geq 3\sqrt[3]{tgAtgBtgC}$
$tgAtgBtgC\geq 3\sqrt[3]{tgAtgBtgC}$ buni har ikkala tomonini kubga oshiramiz:
$(tgAtgBtgC)^3\geq 27tgAtgBtgC$, → $(tgAtgBtgC)^2\geq 27$, →
$tgAtgBtgC=tgA+tgB+tgC\geq 3\sqrt{3}$. Tengsizlik isbotlandi.

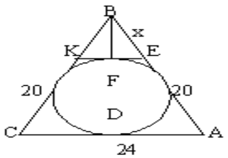
180.

163. A 18 30 B

$\frac{S-30}{v_1}=\frac{30}{v_2}$, $v_2=\frac{30v_1}{S-30}$, $\frac{S-30+18}{v_2}=\frac{60+S-30-18}{v_1}$, $\frac{S-12}{v_2}=\frac{S+12}{v_1}$, $\frac{S-12}{30v_1}(S-30)=\frac{S+12}{v_1}$

S^2-42S+360-30S-360=0, S=72 km.

164. Matematik induksiya metodi yordamida ko'rish mumkinki, ifoda faqat n=1 da 3804 ga bo'linad. Boshqa qiymatlarda bo'linmaydi.
165. S=*ab*
166. BF=y, BE=x, h= $\sqrt{20^2-12^2}=16$, S=(24·16):2=192.
$r=\frac{2S}{a+b+c}=\frac{2\cdot 192}{20+20+24}=6$, y=16-12=4



ΔBEF ~ΔABD , BF=4, x:4=20:16, x=5.
KE=2· $\sqrt{5^2-4^2}$ =6, S_1=(6·4):2=12 $r_1=\frac{2S_1}{5+5+6}=1{,}5$ J: r_1=1,5 sm
167. Ko'rsatma: hosil bo'lgan oltiburchakning yuzi shu uchburchakning o'rta chizig'i va aosoga tushirilgan perpendikulyar hosil qilgan to'g'ri to'rtburchak yuziga tengligini ko'rsating. Shu to'g'ri to'rtburchakning bitta tomoni uchburchakning asosi yarmiga, ikkinchi tomoni esa balandligi yarmiga teng bo'ladi.

168. 10x+y=kxy
1) x=1 da, 10+y=ky, 10=y(k-1)=1·10=2·5=5·2, J: 11,12,13
2) x=2 da 20=y(2k-1)=4·5 J: 24
3) x=3 da 30=y(3k-1)=6·5, J: 36
Tekshirishlar shuni ko'rsatadiki, x ning boshqa qiymatlarida tenglik o'rinli bo'lmaydi. J: 11,12,15,24,36 sonlari.

169. $8\log_a x+\log_x a\le 6$ tengsizlikning yechimi x Є [$\sqrt[4]{a};\sqrt{a}$].
$\cos\left(\frac{\pi x^2}{a^2}\right)\ge\frac{1}{2}$ ni $[-\frac{\pi}{2};\frac{\pi}{2}]$ da ko'ramiz: $0\le\frac{\pi x^2}{a^2}\le\frac{\pi}{3}$, $\frac{\pi x^2}{a^2}\le\frac{\pi}{3}\to\frac{x^2}{a^2}\le\frac{1}{3}$
x= $\sqrt[4]{a}$ da, $a^3\ge 9$, → $a\ge\sqrt[3]{9}$; x= $\sqrt{a}$ da , $a\ge 3$. J: $a\ge 3$.

170. Ko'rsatma: ABCD to'rtburchakning parallelogram ekanligini ko'rsating.

52. 2 km piyoda, 3 km velosipedda, 20 km mototsiklda yurish uchun 1 soat 6 minut ketadi; 5 km piyoda, 8 km velosipedda, 30 km mototsiklda yurish uchun 2 soat 24 minut ketadi. 4 km piyoda, 5 km velosipedda, 80 km mototsiklda yurish uchun qancha vaqt ketadi?

53. Qanday natural n da quyidagi tenglik bajariladi:
$$\sqrt[n]{17\sqrt{5}+38}+\sqrt[n]{17\sqrt{5}-38}=\sqrt{20}$$

54. Yig'indini hisoblang:
$$\left[\frac{x+1}{2}\right]+\left[\frac{x+2}{4}\right]+\left[\frac{x+4}{8}\right]+\dots$$

55. 2 ni verguldan keyin nolga teng bo'lmagan uchta raqami bo'lgan o'nli kasrlarning kvadratlari yig'indisi ko'rinishida yozing.

56. Tenglamani yeching:
$$\cfrac{x}{2+\cfrac{x}{2+\cdots\cfrac{}{2+\cfrac{x}{1+\sqrt{1+x}}}}}=1$$

57. Agar $f(x)=\frac{4^x}{4^x+2}$ bo'lsa, $f\left(\frac{1}{2006}\right)+f\left(\frac{2}{2006}\right)+\dots+f\left(\frac{2005}{2006}\right)$ ni hisoblang.(x Є R)

58. Ifodaning eng kichik qiymatini toping:
$$\frac{a}{b+c}+\frac{b}{c+d}+\frac{c}{d+a}+\frac{d}{a+b}$$ (*a*,*b*,*c*,*d*-musbat sonlar)

59. Agar *ad*-*bc*=1 bo'lsa, $a^2+b^2+c^2+d^2+ac+bd\ge\sqrt{3}$ ni isbotlang.

60. Agar $\frac{x}{y+z+t}=\frac{y}{z+t+x}=\frac{z}{t+x+y}=\frac{t}{x+y+z}$ bo'lsa, $\frac{x+y}{z+t}+\frac{y+z}{t+x}+\frac{z+t}{x+y}+\frac{t+x}{z+t}$ ning qiymatini hisoblang.

61. $a_n=\left\lfloor\sqrt{n(n+2)(n+4)(n+6)}\right\rfloor$ ketma-ketlikning qaysi hadlari 7 ga bo'linadi?

62. $a_n=\sqrt{n(n+2)(n+4)(n+6)}$ ketma-ketlikning qanday hadlari ratsional son bo'ladi?

63. Agar *a*,*b*,*c*,*d*,*e*,*f* >0 bo'lsa, tengsizlikni isbotlang:

$$\frac{a}{b+c}+\frac{b}{c+d}+\frac{c}{d+e}+\frac{d}{e+f}+\frac{e}{f+a}+\frac{f}{a+b}\geq 3$$

64. Tengsizlikni isbotlang: $\sqrt{\frac{a}{b+c}}+\sqrt{\frac{b}{a+c}}+\sqrt{\frac{a}{a+b}}\geq 2$ (*a*,*b*,*c*-musbat sonlar)

65. Ifodaning eng katta va eng kichik qiymatini toping:

$$\frac{(\sin x+\sin y)\cos z+\cos x\cos y\sin z}{1+\sin x\sin y}$$

66. Sonning butun qismini toping:

$$\left(\frac{1989}{1990}\right)^{\frac{1}{\frac{1}{1900}+\frac{1}{1901}+\ldots+\frac{1}{1988}}}$$

67. Agar $\sqrt{\underbrace{\overline{xxx...x}}_{2n}-\underbrace{\overline{yyy...y}}_{n}}=\underbrace{\overline{zzz...z}}_{n}$ tenglik o'rinli bo'lsa, x, y, z raqamlarni toping. (Bu yerda n-ikki xonali son)

68. n ning qanday natural qiymatida kasr qisqaruvchi bo'ladi:

$$\frac{n^4+6n^3+13n^2+12n+3}{n^4+6n^3+15n^2+18n+8}$$

69. Tengsizlikni yeching: $1+2\cdot2^x+3\cdot3^x<6^x$.

70. Qanday n va k natural sonlarda 2^n+1 soni 2^k-1 ga bo'linadi?

71. *a*-musbat son bo'lganda $\sqrt{a^2-a+1},\sqrt{a^2+a+1},\sqrt{4a^2+3}$ tomonli uchburchak mavjudligini isbotlang va uning yuzini *a* ga bog'liq bo'lmagan holda toping.

72. x>π uchun tengsizlik o'rinli ekanligini isbotlang: $\sin x\geq\frac{\pi^2-x^2}{\pi^2+x^2}x$

73. $[\sqrt{n+\alpha}+\frac{1}{2}]=[\sqrt{n}+\frac{1}{2}]$ tenglik n ning ixtiyoriy natural qiymatida o'rinli

t=1 da, z≤4 bo'lgan holda aniq kvadratlarni tekshiramiz:
$\overline{xyzt}$ Є {1521,2401,2601,3721,5041,6241,7921,9801} tekshirishlar shuni ko'rsatadiki masala shartini faqat 3721 qanoatlantiradi.

J: 3,7,2,1 raqamlari

156. J: EK= $\sqrt{7}$

157. cosαcos2αcos4α…$\cos2^n\alpha=\frac{2\sin\alpha\cos\alpha\cos2\alpha...\cos2^n\alpha}{2\sin\alpha}=\frac{\sin2^{n+1}\alpha}{2^{n+1}\sin\alpha}$

158. $(ab-1)^2\geq0$

159. n=2006 deb belgilaymiz va quyidagi tengsizlikni matematik induksiya metodi bilan isbotlaymiz:

$\left(1-\frac{1}{n}\right)\left(1-\frac{1}{n^2}\right)...\left(1-\frac{1}{n^n}\right)\leq\frac{n-2}{n-1}$, n>2

1) n=3 da to'g'ri: $\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{8}{9}\leq\frac{1}{2}$

2) n=k da to'g'ri deb faraz qilamiz: $\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k^2}\right)...\left(1-\frac{1}{k^k}\right)\leq\frac{k-2}{k-1}$

n=k+1 da to'g'riligini tekshiramiz:

$\left(1-\frac{1}{k}\right)\left(1-\frac{1}{k^2}\right)...\left(1-\frac{1}{k^k}\right)\left(1-\frac{1}{k+1^{k+1}}\right)\leq\frac{k-1}{k}\longrightarrow\left(\frac{k-2}{k-1}\right)\left(1-\frac{1}{k+1^{k+1}}\right)\leq\frac{k-1}{k}\longrightarrow 1-\frac{1}{k+1^{k+1}}\leq\frac{(k-1)^2}{k(k-2)}$

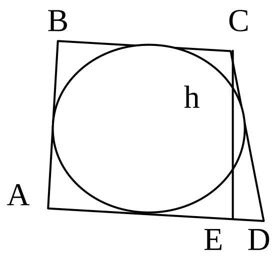
$\frac{k+1^{k+1}-1}{k+1^{k+1}}\leq\frac{k^2-2k+1}{k^2-2k}$, $\frac{k+1^{k+1}-1}{k+1^{k+1}}<1$, $\frac{k^2-2k+1}{k^2-2k}>1$

160. AB=2r, BC=b, AD=a, CD=x bo'lsin. ED=a-b

Δ CED dan $h^2=x^2-(a-b)^2=(x+a-b)(x-a+b)=4(a-r)(b-r)$

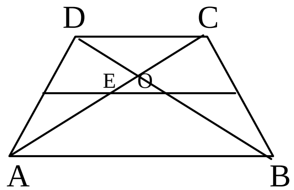
a+b=2r+x , x=a+b-2r, h=2r, $4r^2=4(a-r)(b-r)$

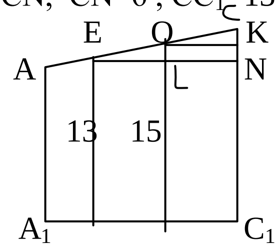
$r=\frac{ab}{a+b}$, $S=\frac{a+b}{2}\cdot2r=\frac{a+b}{2}\cdot2\frac{ab}{a+b}=ab$



161. EKUK(3,4,5,7,11)=4620, 4620+1=4621 J: 4621 ta

162. AE=EC, AO:OC=2:1, AO=2x, OC=x, EO=0,5x
EC=1,5x , AC diagonalni ko'chirib, olib o'tamiz.
ΔOEL ~ ΔECN. OE:OL=EC:CN, 0,5x:2=1,5x:CN, CN=6 , CC_1=13+6=19
AA_1=26-19=7, J: AA_1=7 sm, CC_1=19 sm.





145. Koshi tengsizligi: $m+n\geq 2\sqrt{mn}$ dan foydalanamiz:

$a+b\geq 2\sqrt{ab}, b+c\geq 2\sqrt{bc}, a+c\geq 2\sqrt{ac}$ bu tengsizliklarni hadlab ko'paytiramiz: $(a+b)(b+c)(a+c)\geq 8\sqrt{a^2b^2c^2}=8abc$

146. $h=2h_1$, $DE=1/2AB$, $AB=2DE$. $S_{DEF}=1/2DE\cdot h_1$, $DE\cdot h_1=8$.
$S_{ABC}=1/2\ AB\cdot h=1/2\cdot 2DE\cdot 2h_1=2DE\cdot h_1=16\ sm^2$.

147. J: x=13k-3, y=5-21k.

148. $\overline{abc}+\overline{bca}+\overline{cab}=111(a+b+c)=3\cdot 37(a+b+c)$ bundan $\overline{bca}+\overline{cab}$ ning 37 ga bo'linishi kelib chiqadi.

149. Ko'rsatma: KLMN ning kvadrat yoki romb bo'lishini ko'rsating.

150. J: 7 raqami.

151. $a^3(b+1)-a^2b(b+1)+b^3(a+1)-ab^2(a+1)\geq 0$
$(b+1)(a^2(a-b))+(a+1)(b^2(b-a))\geq 0$
$(a-b)(a^2(b+1)-b^2(a+1))\geq 0$
$(a-b)((a-b)(a+b)+ab(a-b))\geq 0$
$(a-b)^2(a+b+ab)\geq 0$

152. Belgilash kiritamiz: $x^{19}=y$. Tenglama quyidagi ko'rinishga keladi:
$Y+y^5=2y^6$,→ $2y^6-y^5-y=0$, → $y(2y^5-y^4-1)=0$, $y_1=0$, → $x^{19}=0$, $x_1=0$.
$2y^5-y^4-1=0$, → $y^5+y^5-y^4-1=0$, → $(y-1)(2y^4+y^3+y^2+y+1)=0$, $y_2=1$, $x_2=1$
$2y^4+y^3+y^2+y+1>0$, chunki, $y^4>|y^3|$, $y^2>|y|$. J: $x_1=0$, $x_2=1$.

153. 1) x=0 bo'lsin: f(-y)=f(0)+f(y)
2) y=0 bo'lsin: f(x)=f(x)+f(0), → f(0)=0
3) x=y bo'lsin: $f(0)=2f(x)-2x^2$, → $2f(x)=2x^2$, → $f(x)=x^2$. J: $f(x)=x^2$

154. Sonning 3 ga va 44 ga bo'linishini tekshiring. J: (2;3) va (8;9).

155. $32\leq\sqrt{\overline{xyzt}}\leq 99$, $t\leq 3$, $z\leq 4$. Ammo natural sonning kvadrati 2 yoki 3 bilan tugamaydi, demak t=0 yoki 1.

t=0 da, z=0 bo'ladi, $y\geq 5$, $\overline{xy}$ -aniq kvadrat bo'lishi kerak. 1600,2500,3600 va 4900 masala shartini qanoatlantirmaydi.

bo'ladigan α ning barcha qiymatlarini toping.

74. Tub son ikkita bo'luvchiga ega: tub sonning o'zi va 1. Qanday sonlar uchta bo'luvchiga ega?

75. $2^{99}+2^9$ ning 41 ga bo'linishini isbotlang.

76. Ifodani soddalashtiring: $(a+b)(a^2+b^2)(a^4+b^4)\ldots(a^{64}+b^{64})$

77. Agar $a+b+c=0$ bo'lsa, $a^3+b^3+c^3=3abc$ ni isbotlang.

78. $(1+2+2^2)(1+2^3+2^6)(1+2^9+2^{18})(1+2^{27}+2^{54})$ ni hisoblang.

79. Ifodani soddalashtiring: $\left(1+\frac{1}{3}\right)\left(1+\frac{1}{3^2}\right)\left(1+\frac{1}{3^4}\right)\ldots\left(1+\frac{1}{3^{32}}\right)$

80. Agar $a+b+c=1$ va a,b,c>0 bo'lsa, $a^2+b^2+c^2\geq\frac{1}{3}$ ni isbotlang.

81. Ifodani soddalashtiring: $A=\left(3^{2^0}+1\right)\left(3^{2^1}+1\right)\left(3^{2^2}+1\right)\ldots\left(3^{2^n}+1\right)$

82. Ifodani soddalashtiring: $A=(1+b)(1+b^2)(1+b^4)\ldots(1+b^{2^n})$

83. Agar $abcd=1$ va $a,b,c,d>0$ bo'lsa,
$a^2+b^2+c^2+d^2+ab+bc+cd+da+ac+bd\geq 10$ ni isbotlang.

84. Tenglamani yeching: [x]+[2x]+[3x]=3. (bu yerda []-sonning butun qismi)

85. Sonlarni taqqoslang: $\frac{10^{2005}+1}{10^{2006}+1}$ va $\frac{10^{2006}+1}{10^{2007}+1}$

86. $7+77+777+\ldots+\underbrace{777..77}_{n}$ ni hisoblang.

87. $1\cdot 1!+2\cdot 2!+3\cdot 3!+\ldots+n\cdot n!$ ni hisoblang. $(n!=1\cdot 2\cdot 3\cdot\ldots\cdot n)$

88. Ifodaning qiymatini hisoblang: $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\ldots+\frac{1}{\sqrt{2006}+\sqrt{2007}}$

89. 2^{100} necha xonali son bo'ladi?

90. Isbotlang: $10^{30}<2^{100}<10^{31}$.

91. Ixtiyoriy butun sonning kvadrati ikkita 5 bilan tugashi mumkin emasligini isbotlang.

92. Ixtiyoriy uchburchak uchun $\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{1}{r}$ tenglik bajarilishini isbotlang. (bu yerda h_a, h_b, h_c-balandliklar; r-ichki chizilgan aylana radiusi)

93. $7^{9^{\cdot^{\cdot^{\cdot^{9}}}}}$ ning oxirgi ikkita raqamini toping.

94. $1,\frac{1}{2},\frac{1}{2^2},\frac{1}{2^3},...,\frac{1}{2^{2006}}$ sonlari orasiga "+" va "-" ishoralarini qo'yib nol hosil qilish mumkinmi?

95. Tengsizlikni isbotlang: $\sqrt{20+\sqrt{20+\sqrt{20+...+\sqrt{20}}}}<5$

96. 1, 2, 5 tiynlik yordamida 20 tiyinni necha xil usulda maydalash mumkin?

97. Musbat a, b, c sonlari uchun quyidagi tengsizlik bajarilishini isbotlang:

$$\sqrt{a^2-ab+b^2}+\sqrt{b^2-bc+c^2}\geq\sqrt{a^2+ac+c^2}$$

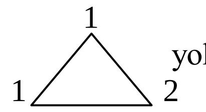
98. Tenglamani butun sonlarda yeching: 60x-77y=1

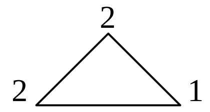
99. Barcha shunday f(x) funksiyalarni topingki, x·f(y)+y·f(x)=(x+y)·f(x)·f(y) shart bajarilsin.

100. $f(x)=5x^2-2x+7$ va g(x)=8x-2 funksiyalar grafiklari orasidagi eng qisqa masofani toping.

101. Limitni hisoblang: $\lim\limits_{t\to1}\frac{t^t-1}{t-1}$

133. Tomoni 1 m bo'lgan muntazam uchburchakni tekislikka tashlaymiz. Faraz qilaylik uning bir uchu birinchi xil rangli nuqtaga va ikkinchi uchu ikkinchi xil rangli nuqtaga tushsin. Uchburchakning uchinchi uchi ikkala xil rangli nuqtadan biriga tushadi. Shart bajarildi.



yoki


bo'ladi.

134. Ifodaning aniqlanish sohasi x=$\frac{4}{3}$ nuqta. Bu nuqtada ifoda 2 ga teng.

135. J: S=r^2(14-2,5π)

136. $\frac{b}{a(a+b)}+\frac{c}{(a+b)(a+b+c)}+\frac{d}{(a+b+c)(a+b+c+d)}=\frac{1}{a}-\frac{1}{a+b}+\frac{1}{a+b}-\frac{1}{a+b+c}+\frac{1}{a+b+c}-\frac{1}{a+b+c+d}=\frac{1}{a}-\frac{1}{a+b+c+d}=\frac{b+c+d}{a(a+b+c+d)}$

137. $(x+1)^3-x^3=3x(x+1)+1$, x(x+1)-juft son, shuniing uchun r=1.

138. Ko'rsatma: tengsizlikning har ikkala tomoni 2 ga ko'paytirib, o'ng tomonini chap tomoniga olib o'ting.

139. (x+1)(x-3)(x-7)

140. 120-masaladan foydalaning. J: x=±2

141. J: 2 ta yechim bor.

142. 2^{500} – m xonali va 5^{500} – n xonali bo'lsin .
$10^{m-1}<2^{500}<10^m$, $10^{n-1}<5^{500}<10^n$. Bularni hadlab ko'paytirsak:
$10^{n+m-2}<10^{500}<10^{n+m}$ → n+m-2<500<n+m → n+m=501. J: 501 xonali

143. Aniqlanish sohasi, x Є (1;3); $\log_{(1-x)}(3-x)=a$ desak , $a=\frac{1}{a}$, a=±1.
J: x=2±$\sqrt{3}$

144. Faraz qilaylik 2a+5 va 3a+4 sonlari p ga bo'linsin. U holda
3a+4-(2a+5)=a-1, 2a+5-(a-1)=a+6, a+6-(a-1)=7, p=7 bo'ladi.
2a+5=7n, 3a+4=7m → $\frac{7n-5}{2}=\frac{7m-4}{3}$, 21n-15=14m-8, → 3n-2m=1, 3n-toq, 2m-juft, shuning uchun n-toq, n=2d-1, a=7d-6. (p,n,m,d Є N)

118. Ko’rsatma: uchburchaklar o’xshashligidan foydalaning.
J: 30sm va 51 sm

119. Ko’rsatma: tenglamani har ikkala tomonini kubga ko’taring. J: x=4416.

120. $\left(\sqrt{2-\sqrt{3}}\right)^x=\frac{1}{\left(\sqrt{2+\sqrt{3}}\right)^x}$ dan foydalaning. J: x=±2

121. J: 1

122. J: $S_n=\frac{8}{17}$

123. J: E(y)=[-5;1]U [2;+∞)

124. J: $x_1=2\pi k$, $x_2=\frac{\pi}{2}+\pi k$, k Є Z.

125. 9+180+2700+36000+450000=5888889, 20032004-5888889=14143115
14143115:7=2020445 , 2020445-1=202044**4**. J: 4 raqami.

126. $\frac{\cos^2 x-\sin^2 x+3(\cos^2 x+\sin^2 x)}{\cos x}\ge 4$

$\frac{4\cos^2 x+2\sin^2 x}{\cos x}\ge 4 \longrightarrow \frac{\cos^2 x+1}{\cos x}\ge 2 \longrightarrow (\cos x-1)^2\ge 0$

127. Ko’rsatma: 84-masaladan foydalaning. J: x Є [0,75;1)

128. $\sqrt{x}$ =t belgilash kiritamiz: $t^2-2t+p=0$, D=4-4p=0 → p=1 J: p=1

129. $a^2+b^2+c^2+a^2\ge 2ab+2ac$
$(a-b)^2+(b-c)^2\ge 0$

130. 13 ta o’quvchi 0,1,2,…,12 ta xato qiladi. Yana 13 ta o’quvchi 0,1,2,…,12 ta xato qiladi. 13+13+1=27, 30-27=3 . Qolgan 3 ta o’quvchi 0,1,2,…12 sonlaridan biricha xato qiladi. 3 ta o’shanday o’quvchi topildi.

131. y>0 da, x=1; y<0 da, x=-1; y=0 da x Є (-∞;+∞)

132. Grafiga O(0;0) nuqtadan iborat.

102. Qanday n Є Z larda $n^5-18n^3-9n^2+13n+24$ va $n^5+3n^4-9n^3+8n+26$ lar bir vaqtda 49 ga bo’linadi?

103. Agar $\frac{x}{x^2+x+1}=a$ bo’lsa, $\frac{x^2}{x^4+x^2+1}$ ni hisoblang.

104. Agar $a+b+c=0$ va $a^3+b^3+c^3=0$ bo’lsa, $a^n+b^n+c^n$ ni hisoblang. (n-toq son, a,b,c Є Q)

105. Agar x Є [-1;1] da $|ax^2+bx+c|\le h$ bo’lsa, $|a|+|b|+|c|\le 4h$ ni isbotlang.

106. Qaysi biri katta $e^e\pi^\pi$ mi yoki $e^{2\pi}$?

107. Tengsizlikni isbotlang: $\log_6 7+\log_7 8+\log_8 9<3{,}3$.

108. Agar α>1, β>1, γ>1 va $\frac{\alpha}{\beta}\ge\frac{\gamma}{\alpha}$ bo’lsa, $\frac{\lg\alpha}{\lg\beta}\ge\frac{\lg\gamma}{\lg\alpha}$ ni isbotlang.

109. Kub ildizni toping: $\sqrt[3]{2+\sqrt{5}}$

110. Qaysi biri katta, $\sqrt{2+\sqrt{3+\sqrt{2+\ldots}}}$ mi yoki $\sqrt{3+\sqrt{2+\sqrt{3+\ldots}}}$ mi?

111. Agar $a>0$, $b>0$, $c>0$ bo’lsa, tengsizlikni isbotlang:
$$2^{a+b}+2^{b+c}+2^{a+c}<2^{a+b+c+1}+1$$

112. Agar *a, b, c* tomonli uchburchak mavjud bo’lsa, $\sqrt{a},\sqrt{b},\sqrt{c}$ tomonli uchburchak ham mavjudligini isbotlang. Teskari mulohaza to’g’rimi?

113. Agar $R(b+c)=a\sqrt{bc}$ bo’lsa, ABC uchburchakning turini aniqlang.

114. Agar x+y=z+t bo’lsa, $x^2+y^2+z^2+t^2$ ifoda 3 ta sonning kvadratlari yig’indisiga teng ekanligini aniqlang.

Turli yillarda Andijon viloyati va Baliqchi tumani Matematika fan olimpiadalarida o'quvchilarga taklif etilgan masalalar.

115. Tenglamalar sistemasini yeching:

$$\begin{cases} \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10 \\ xy=\frac{1}{6} \\ yz=\frac{1}{15} \end{cases}$$

116. Ifodaning 6 ga bo'linishini isbotlang: n(2n+1)(7n+1) , n Є N.

117. Ko'paytuvchilarga ajrating: $x^3-6x^2+11x-6$

118. Radiuslari 17 sm va 10 sm bo'lgan aylanalar kesishadi. Ularning radiuslari orasidagi masofa 21 ga teng. Ularning umumiy urinmasi va markazlari orqali o'tgan to'g'ri chiziqning kesishish nuqtasidan aylanalar markazlarigacha masofalarni toping.

119. Tenglamani yeching: $\sqrt[3]{54+\sqrt{x}}+\sqrt[3]{54-\sqrt{x}}=\sqrt[3]{18}$

120. Tenglamani yeching: $\left(\sqrt{2-\sqrt{3}}\right)^x+\left(\sqrt{2+\sqrt{3}}\right)^x=4$

121. Ifodani soddalashtiring:

$$\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}$$

122. Ifodaning qiymatini toping: $S_n=\frac{1}{1\cdot3}+\frac{1}{3\cdot5}+...+\frac{1}{15\cdot17}$

123. Funksiyaning aniqlanish sohasini toping: $y=\sqrt[8]{\frac{x^2+4x-5}{x-2}}$

124. Tenglamani yeching: 1+cos2x=2cosx

125. 123456789101112…. yozilgan sondagi 20032004 – o'rindagi raqamni toping.

126. cosx≠0 bo'lsa tengsizlikni isbotlang: $\frac{\cos 2x+3}{\cos x}\geq 4$

107. Avval $\log_n(n+1)>\log_{n+1}(n+2)$ ni isbotlaylik:

$$\frac{\log_{n+1}(n+2)}{\log_n(n+1)}=\log_{(n+1)}(n+2)\cdot\log_{n+1}n<\left(\frac{\log_{(n+1)}(n+2)+\log_{(n+1)}n}{2}\right)^2=\left(\frac{\log_{(n+1)}(n+2)n}{2}\right)^2<1$$

$6^{11}>7^{10}$ chunki, $6^{11}=6\cdot36^5>6\cdot5^5\cdot7^5>18000\cdot7^5>350\cdot50\cdot7^5>7^{10}$, shuning uchun $\log_6 7<1{,}1$ qolganlari ham shundaytopiladi.

$\log_6 7+\log_7 8+\log_8 9<1{,}1+1{,}1+1{,}1=3{,}3$.

108. $\alpha^2\geq\beta\gamma$, $2\lg\alpha\geq\lg\beta+\lg\gamma$, $\lg\alpha>0$, $\lg\beta>0$, $\lg\gamma>0$.
$\lg\beta+\lg\gamma\geq2\sqrt{\lg\beta\cdot\lg\gamma}\rightarrow\lg\alpha\geq\sqrt{\lg\beta\cdot\lg\gamma}$.

109. Ko'rsatma: $\sqrt[3]{2+\sqrt{5}}=a+b\sqrt{5}$ ni kubga ko'tarib a va b ni toping.

J: $\frac{1+\sqrt{5}}{2}$

110. J: 2-son katta

111. Belgilash kiritamiz: $2^a=x$, $2^b=y$, $2^c=z$ va yozamiz:
xy+yz+xz<2xyz+1 , x>1, y>1, z>1.
2xyz-xy-yz-xz+1=(x-1)(y-1)(z-1)+(x-1)(yz-1)+(y-1)(z-1)>0

112. $a+b>c$, $(\sqrt{a}+\sqrt{b})^2=a+b+2\sqrt{ab}>a+b+c\rightarrow\sqrt{a}+\sqrt{b}>\sqrt{c}$.

Teskari mulohaza har doim ham to'g'ri emas. M-n, tomonlari 1,1,$\sqrt{2}$ bo'lgan to'g'ri burchakli uchburchak mavjud, ammo tomonlari $1^2=1$, $1^2=$, $(\sqrt{2})^2=2$ bo'lgan uchburchak mavjud emas.

113. $\frac{a}{2R}=\frac{\frac{b+c}{2}}{\sqrt{bc}}\geq1\rightarrow a=2R,\ b=c.$

J: ABC uchburchak -teng yonli to'g'ri burchakli. ∠A=90º, $b=c$.

114. $x^2+y^2+z^2+t^2=x^2+y^2+z^2+t^2-2x(x+y)-z-t)=3x^2+y^2+z^2+t^2+2xy-2xt-2xz=$
$=(x+y)^2+(x-z)^2+(x-t)^2$

115. $\left(\frac{7}{6};\frac{1}{7};\frac{7}{15}\right)$ va $\left(\frac{1}{2};\frac{1}{3};\frac{1}{5}\right)$

116. Ko'rsatma: n=3k, 3k+1 va 3k+2 da tekshiring.

117. $x^3-6x^2+11x-6=x^3-x^2-5x^2+5x+6x-6=x^2(x-1)-5x(x-1)+6(x-1)=$
$=(x-1)(x^2-5x+6)=(x-1)(x-2)(x-3)$

98. 60x=77y+1, $x=\frac{77y+1}{60}=y+\frac{17y+1}{60}$ J: x=9-77n, y=7-60n

99. 1) x=0 bo'lsin: y·f(0)=y·f(0)·f(y), → f(y)=1
2) y=0 bo'lsin: x·f(0)=x·f(0)·f(x), → f(x)=1
3) x=-y bo'lsin: -y·f(y)+y· (f(-x)=0
y·f(-y)=y·f(y), y=0 da f(y)=0; x=y da f(x)=0
J: f(x)=0 va f(x)=1 funksiyalar.

100. f '(x)=10x_0-2=8 , x_0=1; g '(x)=8. y=5-2+7+8(x-1), →y=8x+2
tgα=8 , sinα=$\frac{8}{\sqrt{65}}$. $h=\frac{1}{2}\sin\alpha=\frac{4}{\sqrt{65}}$.

101. f(x)=x^x funksiyani qaraymiz: x_0=1, $x^x=e^{x\ln x}$, f '(x)=$(e^{x\ln x})'$= $x^x(x\ln x)'=x^x(1+\ln x)$, demak lim=1. J: 1

102. 1-sondan koeffitsiyenti 7 bo'lgan hadlarni chiqarib tashlaymiz:
$n^5-4n^3-2n^2-n+3=(n^2+3n+1)(n-2)(n-4)^2$, $n^2+3n+1\neq 7,0$ chunki D=5
Demak n=7k+2 va 7k+4. Ikkinchi sondan n=7k+4 va n=7k+5 chiqadi.
J: n=7k+4 da, k Є Z.

103. x≠0, a≠0. $\frac{x^4+x^2+1}{x^2}=x^2+\frac{1}{x^2}+1=(x+\frac{1}{x})^2-1$ · $\frac{1}{a}=\frac{x^2+x+1}{x}=x+\frac{1}{x}+1$ bo'lgani uchun: $\frac{x^2}{x^4+x^2+1}=\frac{1}{\left(\frac{1}{a}-1\right)^2-1}=\frac{a^2}{1-2a}$.

104. $c=-(a+b)$, $a^3+b^3+c^3=a^3+b^3-(a^3+b^3+3ab(a+b))=3abc=0$,
a,b,c sonlaridan kamida bittasi nolga teng bo'lishi kerak.M-n $c=0$ bo'lsin:
$a=-b$, $a^n+b^n+c^n=-b^n+b^n=0$.

105. f(x)= ax^2+bx+c funksiyani qaraymiz: M=f(1)=$a+b+c$, N=f(-1)=$a-b+c$
→ 2a=M+N-2c, 2b=M-N. |M|≤h, |N|≤h, |c|=|f(0)|≤h.
2|a|=|M+N-2c|≤|M|+|N|+2|c|≤4h, 2|b|=|M-N|≤|M|+|N|≤2h.
|a|+|b|+|c|≤2h+h+h=4h.

106. f:x → x-πlnx funksiyani qaraymiz:
f '(x)=1-$\frac{\pi}{x}$, x=π da minimum; f(e)>f(π) → e-π>π-πlnπ, e+πlnπ>2π
$e^{e+\pi\ln\pi}>e^{2\pi}$, ↔ $e^e\pi^\pi>e^{2\pi}$

127. Tenglamani yeching: [x]+[2x]+[4x]=4

128. Agar x-2$\sqrt{x}$ +p=0 tenglamaning ildizi bitta bo'lsa, p ni toping.

129. Tengsizlikni isbotlang: $2a^2+b^2+c^2\geq 2a(b+c)$

130. Sinfda 30 ta o'quvchi bor. Yozma ishda bitta o'quvchi eng ko'p 12 ta xato qildi. Qolganlari bundan kam xato qilishdi. Shu sinfda bir xil miqdorda xato qilgan kamida 3 ta o'quvchi topilishini toping.

131. y=x|y| funksiyaning grafigini yasang.

132. Tenglamani grafigini yasang: $(y-x^2)^2+y^2=0$

133. Tekislik ixtiyoriy tartibda ikki xil rangga bo'yalgan. Bir-biridan 1 m uzoqlashgan va bir xil rangli ikkita nuqta topilishini isbotlang.

134. Ifodaning qiymatini toping: $\frac{3x-\sqrt{3x-4}}{\sqrt{4-3x}+2}$

135. r va 3r radiusli aylanalar o'zaro tashqi urinadi. Aylanalar va ularga o'tkazilgan umumiy urinma orasidagi figura yuzini toping.

136. Ifodani soddalashtiring: $\frac{b}{a(a+b)}+\frac{c}{(a+b)(a+b+c)}+\frac{d}{(a+b+c)(a+b+c+d)}$

137. 2 ta ketma-ket natural son kublarining ayirmasini 6 ga bo'lganda 1 qoldiq qolishini isbotlang.

138. Tengsizlikni isbotlang: $a+b+c\geq\sqrt{ab}+\sqrt{ac}+\sqrt{bc}$ $(a,b,c>0)$

139. Ifodani ko'paytuvchilarga ajrating: $x^3-9x^2+11x+21$

140. Tenglamani yeching: $\left(\sqrt{7+4\sqrt{3}}\right)^x+\left(\sqrt{7-4\sqrt{3}}\right)^x=14$

141. Agar 100+10a+b<0 bo'lsa, $x^2+ax+b=0$ tenglama nechta ildizga ega?

142. 2^{500} va 5^{500} sonlari ketma-ket yozilgan. Necha xonali son xosil bo'lgan?

143. Tenglamani yeching: $\log_{(1-x)}(3-x)=\log_{(3-x)}(1-x)$

144. *a* ning qanday qiymatlarida $\frac{2a+5}{3a+4}$ kasr qisqaruvchi bo'ladi?

145. Tengsizlikni isbotlang: $(a+b)(b+c)(a+c)\geq 8abc \quad (a,b,c\geq 0)$

146. ABC uchburchakda DE – o'rta chiziq (DE //AB). AB tomondan F nuqta shunday olinganki, AF=3 sm va $S_{DEF}=4$ sm^2. ABC uchburchakning yuzini toping.

147. Tenglamani butun sonlarda yeching: $21x+13y=2$

148. Uch xonali $\overline{abc}$ soni 37 ga bo'linsa, $\overline{bca}+\overline{cab}$ yig'indi ham 37 ga bo'linishini isbotlang.

149. Tomonining uzunligi 1 ga teng bo'lgan ABCD kvadratning AB, BC, CD va DA tomonlaridan mos ravishda K,L,M,N nuqtalar olingan. Agar AK+LC+CM+NA=2 bo'lsa, KM ⊥ LN ni isbotlang.

150. 12345…. Yozilgan sondagi 2005 – o'rindagi raqamni toping.

151. Tengsizlikni isbotlang: $a^3(b+1)+b^3(a+1)\geq a^2(b+b^2)+b^2(a+a^2),\ a,b\geq 0.$

152. Tenglamani yeching: $x^{19}+x^{95}=2x^{19+95}$

153. Agar x Є R va y Є R bo'lsa, ixtiyoriy x va y larda f(x-y)=f(x)+f(y)-2xy shartni qanoatlantiruvchi barcha f funksiyalarni toping.

154. $\underline{4x87y6}$ soni 132 ga bo'linadi. x va y ni toping.

155. $\sqrt{\overline{xyzt}} = x+y^2+z^3+t^4$ munosabatni qanoatlantiruvchi *x, y, z, t* raqamlarni toping.

156. ABCDEF muntazam oltiburchakning tomoni 1 ga teng. AB va CD tomonlarni davom ettirsak, ular K nuqtada kesishadi. EK ning uzunligini toping.

157. Ifodani soddalashtiring: $\cos\alpha\cos2\alpha\cos4\alpha\ldots\cos2^n\alpha$

91. Berilgan son 5 bilan tugashi kerak:
$N^2=(10m+5)^2=100m^2+100m+25=100m(m+1)+25$. Demak 25 bilan tugashi mumkin.

92. $S=\frac{1}{2}ah_a, \Rightarrow \frac{1}{h_a}=\frac{a}{2S}$, xuddi shunday qolgan tomonlar uchun ham $\frac{1}{h_b}=\frac{b}{2S}, \frac{1}{h_c}=\frac{c}{2S}$ o'rinli. S=pr, ($p=\frac{a+b+c}{2}$) $\rightarrow$ $\frac{1}{r}=\frac{p}{S}=\frac{a+b+c}{2S}$

$$\frac{1}{h_a}+\frac{1}{h_b}+\frac{1}{h_c}=\frac{a}{2S}+\frac{b}{2S}+\frac{c}{2S}=\frac{a+b+c}{2S}=\frac{1}{r}.$$

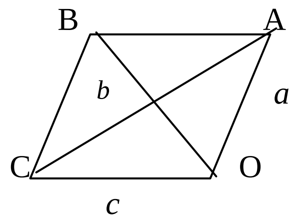
93. $9^{9^{\cdot^{\cdot^{9}}}}=(8+1)^{9^{\cdot^{\cdot^{9}}}}=8m+1$, $7^{8m+1}=(7^4)^{2m}\cdot7=(2401)^{2m}\cdot7=(2400+1)^{2m}\cdot7=$
$=(100k+1)\cdot7=700k+7$ J: 07

94. Sonlarni umumiy maxrajga keltiraylik:
$\frac{2^{2006}}{2^{2006}}, \frac{2^{2005}}{2^{2006}}, \ldots, \frac{1}{2^{2006}}$ bu sonlarning hammasini surati juft, ammo oxirgi kasrning surati toq. Demak bu mumkin emas.

95.
$$\sqrt{20}<5,$$
$$\sqrt{20+\sqrt{20}}<\sqrt{25}=5$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\sqrt{20+\sqrt{20+\sqrt{20+\ldots+\sqrt{20}}}}<5$$

96. J: 29 xil usulda.

97. Geometrik yechilishi:



OA=a, OB=b, OC=c, ∠AOC=120º, ∠AOB=60º ∠BOC=60º. Kosinuslar teoremasiga ko'ra:
$AB^2=OA^2+OB^2-2\cdot OA\cdot OB\cos60^\circ=a^2+b^2-ab$
$BC^2=OB^2+OC^2-2\cdot OB\cdot OC\cos60^\circ=b^2+c^2-bc$
$AC^2=OC^2+AO^2-2\cdot OC\cdot AO\cos120^\circ=a^2+c^2+ac$

AB=$\sqrt{a^2-ab+b^2}$, BC=$\sqrt{b^2-bc+c^2}$, AC=$\sqrt{a^2+ac+c^2}$ Uchburchak tengsizligiga ko'ra: AB+BC>AC :

$$\sqrt{a^2-ab+b^2}+\sqrt{b^2-bc+c^2}\geq\sqrt{a^2+ac+c^2}$$

83. $x+y\ge 2\sqrt{xy}$ va $\left(\sqrt{x}-\sqrt{y}\right)\ge 0$ dan foydalanamiz:
$abcd=1$, $ab+cd\ge 2$, $ac+bd\ge 2$, $ad+bc\ge 2$, $(a^2+b^2)+(c^2+d^2)\ge$
$\ge 2\sqrt{a^2b^2}+2\sqrt{c^2d^2}\ge 4$.
Yuqorida hosil qilinganlarni mos ravishda qo'shsak:
$a^2+b^2+c^2+d^2+ab+bc+cd+da+ac+bd\ge 2+2+2+4=10$.

84. Ko'rinib turibdiki $0\le x<1$. Tenglamani mos ravishda uchta oraliqda yechamiz:
a) $0<x<0,5$: $0+0+1=1\ne 3$
b) $0,5\le x<\frac{2}{3}$: $0+1+1=2\ne 3$
c) $\frac{2}{3}\le x<1$: $0+1+2=3$ J: x Є [$\frac{2}{3}$;1)

85. $10^{2005}=a$ belgilash kiritaylik: $\frac{a+1}{10a+1}\, va\, \frac{10a+1}{100a+1}$.
$(a+1)(100a+1)=100a^2+101a+1>(10a+1)^2=100a^2+20a+1$
J: 1-son katta.

86. $\underbrace{77...77}_{n}=7\cdot\underbrace{11...11}_{n}=\frac{7}{9}\cdot\underbrace{99...99}_{n}=\frac{7}{9}(10^n-1)$ dan foydalanib topamiz:
$\frac{7}{9}(10-1)+\frac{7}{9}(10^2-1)+...+\frac{7}{9}(10^n-1)=\frac{7}{9}(10+10^2+10^3+...+10^n-n)$
$=\frac{7}{9}\left(\frac{10(1-10^n)}{1-10}-n\right)=\frac{7}{9}\left(\frac{10^{n+1}-10}{9}-n\right)$

87. k·k!=(k+1)!-k! dan foydalanamiz:
1·1!+2·2!+3·3!+…+n·n!=2!-1!+3!-2!+4!-3!+…+(n+1)!-n!=(n+1)!-1

88. $\frac{1}{\sqrt{n}+\sqrt{n+1}}=\frac{(\sqrt{n}+\sqrt{n+1})(\sqrt{n+1}-\sqrt{n})}{\sqrt{n}+\sqrt{n+1}}=\sqrt{n+1}-\sqrt{n}$ dan foydalanamiz:
$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+...+\frac{1}{\sqrt{2006}+\sqrt{2007}}\sqrt{2}-1+\sqrt{3}-\sqrt{2}+\sqrt{4}-\sqrt{3}+...+\sqrt{2007}-\sqrt{2006}=\sqrt{2007}-1$

89. k-1≤100lg2<k, dan foydalanamiz: lg2≈0,301. Demak, k=31
J: 31 xonali.

90. $10^3<2^{10}$ buni 10-darajaga ko'taramiz: $10^{30}<2^{100}$;
$2^{13}<10^4$ buni 7-darajaga ko'taramiz: $2^{91}<10^{28}$, va $2^9<10^3$ ni hadlab ko'paytiramiz: $2^{100}<2^{31}$ kelib chiqadi.

158. Tengsizlikni isbotlang: $a^2b^2-2ab+1\ge 0$

159. Tengsizlikni isbotlang: $\left(1-\frac{1}{2006}\right)\left(1-\frac{1}{2006^2}\right)\left(1-\frac{1}{2006^3}\right)...\left(1-\frac{1}{2006^{2006}}\right)\le\frac{2004}{2005}$

160. Aylanaga tashqi chizilgan to'g'ri burchakli trapetsiyaning yuzi uning asoslari ko'paytmasiga tengligini isbotlang.

161. Ota bo'g'dan olmalar keltirdi. Bolalar undan nechta olma keltirganini so'rashdi. Ota sanamaganini, lekn 3 talab , 4 talab, 5 talab, 7 talab, 11 talab qo'yganda har gal 1 tadan olma ortib qolganini aytdi. Ota eng kami bilan nechta olma keltirgan bo'lishi mumkin.

162. Trapetsiyaning asoslaridan biri ikkinchisidan ikki marta katta. Trapetsiyaning o'rta chizig'i α tekislikka parallel va undan 13 sm masofada o'tadi. Trapetsiyaning diagonallarining kesishish nuqtasi esa bu tekislikdan 15 sm masofada yotadi. Trapetsiyaning asoslaridan α tekislikkacha masofalarni toping.

163. 2 ta velosipedchi A va B punktlardan bir-biriga qarab yo'lga chiqdi va B punktga 30 km qolganda uchrashdi. Manzilga yetib qaytdi va A punktga 18 km qolganda uchrashdi. A va B punktlar orasidagi masofani toping.

164. $(n^3-n)\cdot(5^{3n+4}+3^{4n+2})$, n Є N sonning 3804 ga bo'linishini isbotlang.

165. To'g'ri burchakli uchburchakning gipotenuzaga tushirilgan balandligi *a* ga teng va gipotenuzaga tushirilgan medianasi *b* gat eng bo'lsa, shu uchburchakning yuzini toping.

166. Teng yonli uchburchakning yon tomoni 20 ga va asosi 24 ga teng. Shu uchburchakka aylana ichki chizilgan. Shu uchburchakning yon tomonlariga va unga ichki chizilgan aylanaga urinuvchi aylananing radiusini toping.

167. O'tkir burchakli uchburchakni o'rtalaridan qolgan tomonlariga perpendikulyarlar chiqarilgan. Shu perpndikulyarlar ajratgan oltiburchak yuzi uchburchakning yuzining yarmiga tengligini isbotlang.

168. Raqamlari ko'paytmasiga bo'linadigan barcha ikki xonali sonlarni toping.

169. $8\log_a x+\log_x a\le 6$ tengsizlikning yechimlari $\cos\left(\frac{\pi x^2}{a^2}\right)\ge\frac{1}{2}$ tengsizlikning ham yechimlari bo'ladigan a ning barcha qiymatlarini toping.

170. ABCD to'rtburchakda $\angle ABC+\angle BCD=180^0$ va AD=BC bo'lsa , $\angle A=\angle C$ ni isbotlang.

171. Tenglamani yeching: $\sqrt{x-2}+\sqrt{3-x}=x^2-5x+7$

172. $\frac{19n+91}{n+17}$ kasr butun son bo'ladigan barcha n natural sonlarni toping.

173. Uchburchakning medianalaridan yangi ucburchak hosil qilish mumkinligini isbotlang. Shunday uchburchakka misol keltiringki, uning a) bessktrisalaridan; b) balandliklaridan uchburchak yasash mumkin emas bo'lsin.

174. $2006!+\frac{4012!}{2006!}$ sonning 4013 ga qoldiqsiz bo'linishini isbotlang.

175. Ixtiyoriy x, y, z Є(0,1) sonlar uchun x(1-y) + y(1-z) +z(1-x)<1 tengsizlik bajarilishini isbotlang.

176. ABC to'g'ri burchakli uchburchak ichidan O nuqta olingan. $\angle AOB=\angle OBX=\angle OXB=\varphi$ va $\angle A=\alpha$ bo'lsa, $\angle B=90^0$ bo'lsa α ni φ orqali ifodalang.

177. Istalgan n, m Є N va n, m≥2 uchun shunday k (k Є N) mavjud bo'lishini isbotlangki, ular uchun quyidagi tenglik bajarilsin.

$$\left(\frac{n+\sqrt{n^2-4}}{2}\right)^m=\frac{k+\sqrt{k^2-4}}{2}$$

178. n ning qanday eng kichik natural qiymatida $\left[\frac{10^n}{x}\right]=2006$ ([ ]-sonning butun qismi) tenglama butun yechimga ega.

179. Har qanday o'tkir burchakli uchburchak uchun quyidagi tengsizlik bajarilishini isbotlang: $\operatorname{tg}\angle A+\operatorname{tg}\angle B+\operatorname{tg}\angle C\ge 3\sqrt{3}$

180. $ax^2=\sin x$ tenglama aniq n ta ildizga ga. 1)n- toq bo'lganda; 2) n-juft bo'lganda a uchun yuqoridan va quyidan baholashni ko'rsating.

73. n=1 va n=2 da quyidagi tengsizliklarni hosil qilamiz:
$1<\sqrt{1+\alpha}+\frac{1}{2}<2$, $1\le\sqrt{2+\alpha}+\frac{1}{2}<2$, bundan $-3/4\le\alpha<1/4$ kelib chiqadi.
Endi α ning bu qiymatlarida n-ixtiyoriy bo'lganda tengsizlik bajarilishini ko'rsatamiz: $k=[\sqrt{n}+\frac{1}{2}]$ bo'lsin. Bundan,

$$k\le\sqrt{n}+\tfrac{1}{2}<k+1,$$
$$k^2-k+1/4<n<k^2+k+1/4$$
$$k^2-k+1\le n\le k^2+k,$$

$-3/4\le\alpha<1/4$ bo'lgani uchun, $k^2-k+1/4<n+\alpha<k^2+k+1/4$,
Bundan, $[\sqrt{n+\alpha}+\frac{1}{2}]=k$ kelib chiqadi.

74. Tub sonning kvadrati.

75. $2^{99}+2^9=(2^{33})^3+(2^3)^3=(2^{33}+2^3)(2^{66}-2^{36}+2^6)=(2^{11}+2)(2^{22}-2^{12}+2^2)(2^{66}-2^{36}+2^6)$
$2^{11}+2=2050=41\cdot5\cdot10$. Ifoda 41 ga bo'linadi.

76. $(a+b)(a-b)=a^2-b^2$ bo'lgani uchun ifodani (a-b) ga ko'paytirib bo'lamiz:
$(a-b)(a+b)(a^2+b^2)(a^4+b^4)\ldots(a^{64}+b^{64})=\frac{a^{128}-b^{128}}{a-b}$.

77. $b=at$ bo'lsin, $c=-a(1+t)$
$a^3+b^3+c^3=a^3+a^3t^3-a^3(1+t)^3=a^3(1+t^3-1-3t-3t^2-t^3)=3a\cdot at\cdot(-a)(1+t)=3abc$.

78. Ko'rsatma: ifodani (2-1) ga ko'paytiring. J: $2^{81}-1$.

79. Ko'rsatma: ifodani $\left(1-\frac{1}{3}\right)$ ga ko'paytirib, natijani shunga bo'lib qo'ying: J: $\frac{3}{2}\left(1-\frac{1}{3^{64}}\right)$

80. $a^2+b^2+c^2=(a+b+c)^2-2(ab+ac+bc)$
$1-2(ab+ac+bc)\ge\frac{1}{3}$
$2(ab+ac+bc)\le\frac{2}{3}$
$-2(ab+ac+bc)\ge-\frac{2}{3}$. $(a+b+c)^2=1$, $a^2+b^2+c^2\ge 1+(-\frac{2}{3})=\frac{1}{3}$

81. Ko'rsatma: ifodani $1=\frac{1}{2}\left(3^{2^0}-1\right)$ ga ko'paytiring. J: $A=\frac{1}{2}\left(3^{2^{n+1}}-1\right)$

82. Ko'rsatma: ifodani (1-b) ga ko'paytirib natijani shunga bo'ling.
J: $A=\frac{1-b^{2^{2n}}}{1-b}=1+b+b^2+\ldots+b^{2n+1}-1$

$+4(n^2+3n)+3=(n^2+3n+1)(n^2+3n+3)$,

$n^4+6n^3+15n^2+18n+8=(n^4+6n^3+9n^2)+6n^2+18n+8=(n^2+3n)^2+6(n^2+3n)+8=$

$=(n^2+3n+2)(n^2+3n+4)$.

n^2+3n+1 va n^2+3n+3 sonlari n^2+3n+2 ga nisbatan o'zaro tub, shuning uchun ularning n^2+3n+2 bilan umumiy boluvchisi yo'q.

n^2+3n+3 va n^2+3n+4 ham o'zaro tub, shuning uchun agar kasr qisqaradigan bo'lsa, u n^2+3n+1 va n^2+3n+4 larning umumiy bo'luvchisiga qisqaradi. Agar $d\neq 1$ – ularning umumiy bo'luvchisi bo'lsin, demak u 3 ga bo'linadi, masalan d=3 bo'lsin; n^2+1 ning 3 ga bo'linishi kelib chiqadi, ammo bu noto'g'ri. Demak kasr hech qanday n da qisqaruvchi bo'lmaydi.

69. Tengsizlikning har ikkala tomoni 6^x ga bo'lamiz va yozamiz:
$\left(\frac{1}{6}\right)^x+2\left(\frac{1}{3}\right)^x+3\left(\frac{1}{2}\right)^x<1$, chap tomondagi f(x) funksiya kamayuvchi. Shuning uchun f(2)=1, f(x)<1, $\leftrightarrow$ x>2.

70. k=1 da n-ixtiyoriy son bo'lishi mumkin; k>1 da esa n ni k ga qoldiqli bo'lamiz: n=kq+r, bu yerda $0\leq r<k$. $2^n-2^r=2^r(2^{kq}-1)$ shuning uchun 2^n+1 soni 2^k-1 ga bo'linishi uchun 2^r+1 soni 2^k-1 ga bo'linishi kerak.
Ammo, k>2 da $2^r+1\leq 2^{k-1}+1<2^k-1$, k=2 da esa 2^r+1 soni 3 ga bo'linadi, r=1, n-ixtiyoriy toq son.
Javob: (s;1) va (2s-1;2) bu yerda s-ixtiyoriy natural son.

71. Bunday uchburchak mavjud bo'lishi uchun eng katta tomoni qolgan ikkitasining yig'indisidan kichik bo'lishi kerak:
$\sqrt{4a^2+3}<\sqrt{a^2-a+1}+\sqrt{a^2+a+1}$. Bu tengsizlikni kvadratga oshirib, soddalashtirib yangi tengsizlik hosil qilamiz: $2a^2+1<2\sqrt{a^4+a^2+1}$, buni yana bir kvadratga oshirsak to'g'ri tengsizlik hosil bo'ladi, demak uchburchak mavjud.
Kosinuslar teoremasiga ko'ra katta burchakning kosinusini topamiz:
$\cos\varphi=\frac{2a^2+1}{2\sqrt{a^2-a+1}\sqrt{a^2+a+1}}=-\frac{2a^2+1}{2\sqrt{a^4+a^2+1}}$, bu burchak sinusi esa:
$\sin\varphi=\frac{\sqrt{3}}{2\sqrt{a^4+a^2+1}}$, uchburchak yuzini topamiz:
$S=\frac{1}{2}\sqrt{a^2+a+1}\sqrt{a^2-a+1}\sin\varphi=\frac{\sqrt{3}}{4}$. Demak, uchburchak mavjud va $S=\frac{\sqrt{3}}{4}$.

72. Belgilash kiritamiz: $x=y+\pi$;
$$\sin x+\frac{x^2-\pi^2}{x^2+\pi^2}x=-\sin y+\frac{y^2+2\pi y}{y^2+2\pi y+2\pi^2}(y+\pi)=-\sin y+\frac{y^3+3\pi y^2+2\pi^2 y}{y^2+2\pi y+2\pi^2}=-\sin y+y+\frac{\pi y^2}{y^2+2\pi y+2\pi^2}>0$$
bunda y>0, y>0 da esa siny<y.

Javoblar, yechimlar, ko'rsatmalar

1. Agar x va y tenglama shartini qanoatlantirsa, ular $x=10z^2$, $y=10t^2$ (bu yerda z va t natural sonlar) ko'rinishda bo'ladi.
$\frac{1}{z}+\frac{1}{t}=\frac{1}{10}$ tenglamaga keladi. (z-10)(t-10)=100, z=10+d desak, $t=10+\frac{100}{d}$ bo'ladi. Tenglama 9 ta butun yechimga ega.

2. $a^{\log_c b}=b^{\log_c a}$ dan foydalanamiz.
$9^{\log_2 x}=x^2\cdot 3^{\log_2 x}-x^{\log_2 3}$
$3^{\log_2 x}=x^2-1$, $\log_2 x=y$ deb belgilaylik, $3^y+1=4^y$ yoki,
$\left(\frac{3}{4}\right)^y+\left(\frac{1}{4}\right)^y=1$ y=1 yagona yechim. Demak, x=2

3. $19^{50}=(20-1)^{50}=1000A-\frac{50\cdot 49}{2}\cdot 20^2+20\cdot 50+1$. Bundan 19^{50} ni 1000 ga bo'lganda 1 qoldiq qoladi. Boshqa tomondan,
$8^7=2^{21}=2\cdot 2^{20}=2\cdot 1024\cdot 1024=50B+2\cdot 24\cdot 24=50C+2$
Berilgan sonni 1000 ga bo'lganda $19^2=361$ qoldiq qoladi.
J: 361

4. $\bar{a}=(x^2,y^2,z^2)$ va $\bar{b}=(1,1,2)$ vektorlarni ko'ramiz: $|\bar{b}|=\sqrt{6}$. Shartdan $|\bar{a}|=1$, $\bar{a}\bar{b}=\sqrt{7}$ kelib chiqadi. $\bar{a}\bar{b}>|\bar{a}|\cdot|\bar{b}|$ xosil bo'lmoqda, lekin bu mumkin emas. Tenglamalar sistemasi yechimga ega emas.

5. Koshi tengsizligidan foydalanamiz: $2^{\sin x}+2^{\operatorname{tg}x}\geq 2\sqrt{2^{\sin x+tgx}}$.
$\left(0,\frac{\pi}{2}\right)$ oraliqda $f(x)=\sin x+\operatorname{tg}x-2x\geq 0$ o'rinli, chunki,
$f'(x)=\cos x+\frac{1}{\cos^2 x}-2>\cos x+\frac{1}{\cos x}-2\geq 0$

6. (a,b) tenglama yechimi bo'lsin.
$\left(a-b\sqrt{3}\right)\left(a+b\sqrt{3}\right)=1$, $\rightarrow$ $\left(a-b\sqrt{3}\right)^2\left(a+b\sqrt{3}\right)^2=1$
$\left(a^2+3b^2-2ab\sqrt{3}\right)\left(a^2+3b^2+2ab\sqrt{3}\right)=1$
$(a^2+3b^2)^2-3(2ab)^2=1$ $(a^2+3b^2;2ab)$ ham tenglama yechimi. (2,1) ham yechim. Tenglama cheksiz ko'p yechimga ega.

7. Birinchi yechim: har qanday $(m^2;0;m)$ butun sonlar yechim bo'la oladi.
Ikkinchi yechim: (a,b,c) tenglama yechimi bo'lsa, (k^6a,k^4b,k^3c) ham

yechim bo'ladi.
Masalan: $1+2^3=3^2$ bo'lgani uchun $3^6+9^3\cdot 2^3=3^8$ o'rinli. (27,18,9) ham yechim.

8. Tenglamadan quyidagini topamiz:
$x^2+y^2-2xy+2x^2y^2-x^3y-xy^3=0$ yoki $(x-y)^2(1-xy)=0$ bundan x=y va xy=1 bo'ladi. Grafik esa to'g'ri chiziq va giperboladan iborat.

9. $x^2+y^2+z^2+t^2 = x^2+y^2+z^2+t^2+2x(x+y-z-t)= 3x^2+y^2+z^2+t^2 +2xy-2xz-2xt= =(x+y)^2+(x-z)^2+(x-t)^2$

10. Tenglamani quyidagich yozamiz:
$\frac{12}{\sqrt{x^3+24}+\sqrt{x^3+12}}=3x+8$ tenglamaning chap qismi kamayuvchi va o'ng qismi esa o'suvchi funksiya. Demak tenglama biita yechimga ega, x=-2

11. Har bir qavs ichini 16 ga ko'paytiramiz va $n^4+4=(n^2-2n+2)(n^2+2n+2)= =((n-1)^2+1)((n+1)^2+1)$ dan foydalanamiz.
$$\frac{(2^4+4)(6^4+4)...(38^4+4)}{(4^4+4)(8^4+4)...(40^4+4)}=\frac{(1^2+1)(3^2+1)(5^2+1)...(37^2+1)(39^2+1)}{(3^2+1)(5^2+1)(7^2+1)...(39^2+1)(40^2+1)}=\frac{1}{841}$$

12. Koshi tebgsizligidan topamiz:
$$\frac{x}{y}+\frac{1}{2}\sqrt{\frac{y}{z}}+\frac{1}{2}\sqrt{\frac{y}{z}}+\frac{1}{3}\sqrt[3]{\frac{z}{x}}+\frac{1}{3}\sqrt[3]{\frac{z}{x}}+\frac{1}{3}\sqrt[3]{\frac{z}{x}}\geq 6\sqrt[6]{\frac{x}{y}\cdot\frac{y}{z}\cdot\frac{z}{x}\cdot\frac{1}{4}\cdot\frac{1}{27}}=\sqrt[6]{2^4\cdot 3^3}>2$$

13. $(x\sqrt{1+y^2}+y\sqrt{1+x^2})^2=x^2(1+y^2)+y^2(1+x^2)+2xy\sqrt{(1+x^2)(1+y^2)} =$
$=(1+x^2)(1+y^2)+x^2y^2+2xy\sqrt{(1+x^2)(1+y^2)}-1=(xy+\sqrt{(1+x^2)(1+y^2)})^2-1$
Istalgan ifoda $\sqrt{a^2-1}$ yoki $-\sqrt{a^2-1}$ ga teng.

14. Berilgan tengsizlikni $n^4>(n+1)^3$ ko'rinishda yozishimiz mumkin.
$n^4-(n+1)^3=n^4-n^3-3n^2-3n-1=n^3(n-3)+2n^2(n-3)+3n(n-3)+6(n-3)+17>0$

15. Kosinuslar teoremasiga ko'ra, $2ab\cos C=a^2+b^2-c^2$, $2bc\cos A=b^2+c^2-a^2$, $2ac\cos B=a^2+c^2-b^2$ bu tengliklarni qo'shib va masala shartidan foydalanib, $a^2+b^2=c^2$ ni topamiz. Bundan ko'rinadiki uchburchak to'g'ri burchakli.

16. Ixtiroriy uchburchak uchun $\mathrm{tg}A+\mathrm{tg}B+\mathrm{tg}C-\mathrm{tg}A\cdot\mathrm{tg}B\cdot\mathrm{tg}C=0$ o'rinli.

ayirma $(k+4)^2-m^2\geq 121-100=21$, 21>16. Demak ketma-ketlikning hech qanday hadi ratsional bo'la olmaydi.

63. Tengsizlikning chap tomonini S bilan belgilaymiz, Koshi-Bunyakovskiy tengsizligiga ko'ra,
$S(a(b+c)+b(c+d)+c(d+e)+d(e+f)+e(f+a)+f(a+b)\geq(a+b+c+d+e+f)^2$, yoki
$S((a+d)(b+e)+(b+e)(f+c)+(f+c)(a+d))\geq(a+b+c+d+e+f)^2$,
Yana belgilash kiritamiz, $a+d=p,\ b+e=q,\ f+c=r,\rightarrow S(pq+qr+pr)\geq(p+q+r)^2$.
Ammo $(p+q+r)^2=p^2+q^2+r^2+2(pq+pr+qr)\geq 3(pq+pr+qr)$,
shuning uchun $S\geq 3$.

64. $a+b+c=t$ deb belgilaymiz va Koshi tengsizligidan foydalanamiz:
$\sqrt{\frac{b+c}{a}}\leq\frac{\frac{b+c}{a}+1}{2}=\frac{t}{2a}$ shuning uchun $\sqrt{\frac{a}{b+c}}\geq\frac{2a}{t}$. Buni har bir hadga qo'llasak,
$\sqrt{\frac{a}{b+c}}+\sqrt{\frac{b}{a+c}}+\sqrt{\frac{c}{a+b}}\geq\frac{2a}{t}+\frac{2b}{t}+\frac{2c}{t}=2.$

65. $(\sin x+\sin y)^2+(\cos x\cos y)^2=\sin^2x+\sin^2y+2\sin x\sin y+(1-\sin^2x)(1-\sin^2y)= =1+2\sin x\sin y+\sin^2x\sin^2y=(1+\sin x\sin y)^2$. Berilgan A ifodani
$\sin\varphi\cos z+\cos\varphi\sin z=\sin(\varphi+z)$ ko'rinishda yozish mumkin va $-1\leq A\leq 1$.
Bundan tashqari, x=y=0, $z=\frac{\pi}{2}\rightarrow A=1$; $x=y=\frac{\pi}{2}$, $z=\pi\rightarrow A=-1$.
Demak -1 va 1 ifodaning eng kichik va eng katta qiymatlaridir.

66. Berilgan sonni A bilan va darajadagi kasr maxrajini p bilan belgilaylik,
$\frac{89}{1988}<p<\frac{89}{1900}$ bundan, $\frac{1900}{89}<\frac{1}{p}<\frac{1989}{89}$ va Bernulli tengsizligidan foydalanamiz:
$A=\left(1+\frac{89}{1900}\right)^{\frac{1}{p}}>1+\frac{89}{1900}\cdot\frac{1}{p}>2$. Shuningdek,
$A=\left(1+\frac{89}{1900}\right)^{\frac{1}{p}}<\left(1+\frac{89}{900}\right)^{\frac{1989}{89}}=\left(1+\frac{89}{1900}\right)\left(1+\frac{89}{1900}\right)^{\frac{1900}{89}}<\left(1+\frac{1}{20}\right)e<3$. Demak, [A]=2.

67. Berilgan tenglikni quyidagicha yozamiz:
$x\frac{10^{2n}-1}{9}-y\frac{10^n-1}{9}=z^2\left(\frac{10^n-1}{9}\right)^2$ yoki, $10^n(9x-z^2)=9y-9y-z^2$.
Agar $9x-z^2\neq 0$, ikki xonali n uchun tenglik bajarilmaydi, shuning uchun $9x=z^2$, $9y=9x+z^2$, bundan y=2x kelib chiqadi.
Demak, x=1, y=2, z=3; yoki x=4, y=8, z=6.

68. Topamiz: $n^4+6n^3+13n^2+12n+3=(n^4+6n^3+9n^2)+4n^2+12n+3=(n^2+3n)^2+$

56. $\sqrt{x+1}-1=\frac{x}{\sqrt{x+1}+1}$ dan foydalansak, x≥-1 da ko'p qavatli kasr $\sqrt{x+1}-1$ga teng. $\sqrt{x+1}-1$=1 ni yechsak x=3 kelib chiqadi.

57. Agar x+y=1 bo'lsa, f(x)+f(y)= $\frac{4^x}{4^x+2}+\frac{4^y}{4^y+2}=\frac{4+2\cdot 4^x+4+2\cdot 4^y}{4+4+2(4^x+4^y)}=1$

Izlanayotgan yig'indi 1002+f($\frac{1}{2}$)=1002 $\frac{1}{2}$ gat eng.

58. x,y>0 uchun quyidagi tengsizlik o'rinli:

$(x+y)^2=x^2+2xy+y^2\geq 4xy$, $\frac{1}{xy}\geq\frac{4}{(x+y)^2}$. Shuning uchun,

$$S=\left(\frac{a}{b+c}+\frac{c}{d+a}\right)+\left(\frac{b}{c+d}+\frac{d}{a+b}\right)=\frac{a(d+a)+c(b+c)}{(b+c)(d+a)}+\frac{b(a+b)+d(c+d)}{(c+d)(a+b)}\geq 4\frac{a^2+ad+bc+c^2}{(a+b+c+d)^2}+4\frac{b^2+ab+cd+d^2}{(a+b+c+d)^2}=$$

$$=2\frac{(a+b+c+d)^2+(a-c)^2+(b-d)^2}{(a+b+c+d)^2}\geq 2$$

Demak, S=2, masalan *a=b=c=d* bo'lsa, *b+c=d+a, c+d=a+b, a=c, b=d, a=d, b=c* bo'ladi.

59. $(ad-bc)^2+(ac+bd)^2=(a^2+b^2)(c^2+d^2)$, shuning uchun

S= $a^2+b^2+c^2+d^2+ac+bd\geq\sqrt{2(a^2+b^2)(c^2+d^2)}$ +(ac+bd)=2 $\sqrt{(ac+bd)^2+1}$ +(ac+bd).

Ammo, $(2\sqrt{x^2+1}+x)^2=4x^2+4+4x\sqrt{x^2+1}+x^2=(2x+\sqrt{x^2+1})^2+3\geq 3$.

60. Berilgan tenglikning har biriga 1 ni qo'shamiz va yozanmiz:

$\frac{x+y+z+t}{y+z+t}=\frac{x+y+z+t}{z+t+x}=\frac{x+y+z+t}{t+x+y}=\frac{x+y+z+t}{x+y+z}$. Agar kasrlarning qiymatlari 0 ga teng bo'lmasa osongina ko'rish mumkinki, x=y=z=t va ifoda 4 ga teng bo'ladi; Agar kasrlar nolga teng bo'lsa, Ifodaning qiymati -4 ga teng bo'ladi.

61. Ildiz ostidagi birinchi ko'paytuvchini to'rtinchi ko'paytuvchi bilan va ikkinchini uchinchisi bilan ko'paytiramiz va **n^2+6n** ni **k** bilan belgilasak, ildiz ostidagi ifoda k^2+8k ko'rinishga keladi. Ammo,

$k^2+6k+9<k^2+8k<k^2+8k+16$, $[\sqrt{k^2+8k}]$=k+3, shuning uchun

$a_n=n^2+6n+3=(n+3)^2-6$. Endi a_n had 7 ga bo'linishi uchun $(n+3)^2$ ni 7 ga bo'lganda 6 qoldiq qolishi kerak, lekin natural sonning kvadratini 7 ga bo'lganda 0,1,4,2 qoldiq qoladi. Demak, a_n hech qanday n da 7 ga bo'linmaydi.

62. 61-masaladan foydalansak, ildiz ichi k^2+8k ga teng. Shartga ko'ra $k^2+8k=m^2$, → $(k+4)^2-m^2=16$. Ammo, $k=n^2+6n\geq 7$, k+4≥11, eng kichik

(chunki tgA- tgA·tgB·tgC=-tg(B+C)(1- tgB·tgC)= $-\frac{tgB+tgC}{1-tgB\cdot tgC}$ (1- tgB·tgC) =-tgB-tgC) tgB va tgC ni tgA orqali shatdan ifodalaymiz va topamiz:
tgA=1, tgB=2, tgC=3

Keyin esa, *a:b:c*=sinA:sinB:sinC= $\frac{tgA}{\sqrt{1+tg^2A}}:\frac{tgB}{\sqrt{1+tg^2B}}:\frac{tgC}{\sqrt{1+tg^2C}}=\sqrt{5}:2\sqrt{2}:3$

17. $\alpha=\sqrt{2}$ va $\beta=\log_{\sqrt{2}}3$ bo'lsin. Ko'rinib turibdiki, β-irratsional, chunki $\log_{\sqrt{2}}3=\frac{p}{q}$ bo'lsin, p va q natural sonlar. $3^q=\sqrt{2}^p$, $9^q=2^p$ bu mumkin emas. Boshqa tomondan, $\alpha^\beta=\sqrt{2}^{\log_{\sqrt{2}}3}$=3.

18. Topamiz:

$\frac{x^5-y^5}{x-y}<\frac{x^5+y^5}{x-y}=1$, $x^4+y^4+x^3y+x^2y^2+xy^3<1$, $x^4+y^4<1$

19. Agar sinx Є Z bo'lsa, sinx=0 tenglamaga keladi va x=πk (k Є Z) bo'ladi. Agar 0<sinx<1 bo'lsa ham [sinx]=0 bo'ladi va avvalgi ko'rinishga keladi, ammo bu holda yechim yo'q. Agar -1<sinx<0 bo'lsa, [sinx]=-1, {sinx}=sinx+1. Tenglama sinx=-1/2 bo'ladi va yechim
$x=(-1)^{k+1}\frac{\pi}{6}+\pi k$ (k Є Z) .

20. [x]≤x va x>0 uchun $\frac{1}{[x]}\geq\frac{1}{x}$ bo'ladi, bundan tenglama musbat sonlarda yechimga ega emas.

x<0 bo'lsin, [x] va {x} ni y va z bilan belgilaylik. Quyidagini topamiz: $\frac{1}{y}+\frac{1}{z}=-\frac{1}{y+z}$, yoki, $y^2+3yz+z^2=0$, bundan $z=\frac{-3+\sqrt{5}}{2}y$ yoki $z=\frac{-3-\sqrt{5}}{2}y$ ni topamiz. Ammo |y|≥1, |z|<1 va shuning uchun ikkinchi tenglik o'rinli emas, demak $z=\frac{-3+\sqrt{5}}{2}y$. 0<z<1 da $\frac{2}{\sqrt{5}-3}<y<0$ bundan $y_1=-1$ va $y_2=-2$

$z_1=\frac{3-\sqrt{5}}{2}$ va $z_2=3-\sqrt{5}$. Tenglamaning ildizi $x_1=y_1+z_1=\frac{1-\sqrt{5}}{2}$ va $x_2=z_2+y_2=1-\sqrt{5}$.

21. t= $\frac{1}{2}\log_4 x$ bo'lsin, $x=16^t$ bo'ladi. Tenglamani $4^t+2^t=6^t$ ko'rinishda yoki $\left(\frac{2}{3}\right)^t+\left(\frac{1}{3}\right)^t=1$ va bundan t=1 , x=16 kelib chiqadi. J: x=16

22. Topamiz: 9-8sin50º=9+8cos80º-8cos80º-8cos40º= =9+8sin10º-16cos60ºcos20º=9+8sin10º-8cos20º=1+8sin10º+8(1-cos20º)= $=1+8\sin 10^\circ+16\sin^2 10^\circ=(1+4\sin 10^\circ)^2$ demak, a=1, b=4, c=10

23. Berilgan tengsizlikning chap tomonidagi 38,36,34,… sonlarni 40 bilan almashtiraylik: $\sqrt{40\sqrt{40\sqrt{40...\sqrt{40\sqrt{40}}}}}=40^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+...+\frac{1}{2^{20}}}<40$

24. $x^4-4x^3-1=(x^2-2x-1)^2-2(x+1)^2$ berilgan tenglama quyidagi ko'rinishga keladi:

$$(x^2-(2+\sqrt{2})x-(1+\sqrt{2}))(x^2-(2-\sqrt{2})x-(1-\sqrt{2}))=0$$

Chap tomondagi ifodadan ikkinchi qavsning diskirminanti $(2-\sqrt{2})^2+4(1-\sqrt{2})=10-8\sqrt{2}$ bu manfiy;

tenglam ikkita yechimga ega: $x_{1,2}=\frac{2+\sqrt{2}\pm\sqrt{10+8\sqrt{2}}}{2}$

25. x^2 ni y bilan belgilaylik. Chap tomondagi funksiya y ga nisbatan o'suvchi funksiya, demak u bittadan ortiq yechimga ega bo'lmaydi, korinib turibdiki $y=a^2+b^2+c^2$ uning yrchimi bo'ladi.

Tenglama $x_{1,2}=\sqrt{a^2+b^2+c^2}$ ildizga ega.

26. Agar n=100k+6 bo'lsa, n^2 soni 36 bilan tugaydi. Bundan tashqari 2^{20} soni 76 bilan, 76 ning ixtiyoriy darajasi yana 76 bilan, 76·64 soni 64 bilan tugaydi.Shuning uchun:

$2^n=2^{100k+6}=(2^{20})^{5k}\cdot 64=(100m+76)\cdot 64$ ifoda 64 bilan tugaydi, 2^n+n^2 ifoda ikkita nol bilan tugaydi va 100 ga bo'linadi.

J: cheksiz ko'p

27. Ifodani a_k bilan belgilaymiz: $\sqrt{k\sqrt{(k+1)...\sqrt{(n-1)\sqrt{n}}}}$

$a_k \geq k+1$ ni ismotlaymiz. $a_k=\sqrt{ka_{k+1}}$, $a_{k+1}=\frac{a_k^2}{k}\geq\frac{k^2+2k+1}{k}>k+2$

Shuning uchun agar $a_2\geq 3$ bo'lsa, $a_{n-1}\geq n$, $\sqrt{(n-1)\sqrt{n}}\geq n$ noto'g'ri tengsizlik.

Demak, $a_2<3$, isbotlandi.

28. Belgilash kiritamiz, $y_n=x_n-1$, topamiz: $y_{n+1}=y_n^2$ va shuning uchun $x_{10}=x_1 \leftrightarrow y_{10}=y_1 \leftrightarrow y_1^{512}=y_1 \leftrightarrow (y_1=1$ yoki $y_1=0)$

Demak $x_{10}=x_1$ bo'lishi uchun $x_1=1$ yoki $x_1=2$ bo'lishi kerak.

$\begin{cases}2x+3y+20z=66\\5x+8y+30z=144\end{cases}$ yoki, $\begin{cases}2x+3y=66-20z\\5x+8y=144-30z\end{cases}$

bundan topamiz: $\begin{cases}x=96-70z\\y=-42+40z\end{cases}$

va shart bo'yicha 4x+5y+80z=4(96-70z)+5(40z-42)+80z=174

J: 2 soat 54 minut ketadi.

53. Birinchi qo'shiluvchini a bilan belgilaylik, tenglik quyidagi ko'rinishga keladi: $a+\frac{1}{a}=\sqrt{20}$ bundan $a=\sqrt{5}+2$ kelib chiqadi. $(\sqrt{5}+2)^3=17\sqrt{5}+38=(\sqrt{5}+2)^3$

Demak, n=3.

54. Qandaydir n-nomerdan keyin $\frac{x}{2^n}$ modul bo'yicha $\frac{1}{2}$ dan kichik bo'ladi va $\frac{x}{2^n}+\frac{1}{2}$ ifoda 0 va 1 ning oralig'ida bo'ladi, demak uning butun qismi 0 ga teng. $\left[x+\frac{1}{2}\right]$=[2x]-[x] bo'lgani uchun (0≤x<1/2 va 1/2≤x<1 da ko'rish mumkin). Ifodaning har bir hadini quyidagicha yoza olamiz:

$$\left[\frac{x}{2}+\frac{1}{2}\right]=[x]-\left[\frac{x}{2}\right]$$
$$\left[\frac{x}{4}+\frac{1}{2}\right]=\left[\frac{x}{2}\right]-\left[\frac{x}{4}\right]$$
$$\left[\frac{x}{8}+\frac{1}{2}\right]=\left[\frac{x}{4}\right]-\left[\frac{x}{8}\right]$$
$$\ldots\ldots\ldots\ldots\ldots\ldots$$
$$\left[\frac{x}{2^n}+\frac{1}{2}\right]=\left[\frac{x}{2^{n-1}}\right]-\left[\frac{x}{2^n}\right]$$

Bundan $\left[\frac{x}{2}+\frac{1}{2}\right]+\left[\frac{x}{4}+\frac{1}{2}\right]+...+\left[\frac{x}{2^n}+\frac{1}{2}\right]=[x]-\left[\frac{x}{2^n}\right]$, n yetarli darajada katta bo'lsa $\left[\frac{x}{2^n}\right]$ ifoda x>0 da 0 ga teng; x<0 da -1 ga teng.

Demak berilgan yig'indi x>0 da [x] ga; x<0 da [x]-1 ga teng.

55. Biz x va y natural sonlarni shunday topamizki, $2000000=x^2+y^2$ bo'ladi. Buning uchun 2000000 soni ikkita $x+yi$ va x-yi kompleks sonlarning ko'paytmasi shaklida yozishimiz kerak.

$2000000=2^7\cdot 5^6=(1+i)^7(2+i)^6(1-i)^7(2-i)^6$,

$(1+i)^7=(1+i)^6(1+i)=(2i)^3(1+i)=8-8i$

$(2+i)^6=(3+4i)^3=27+108i-144-64i=-117+44i$

$(1+i)^7(2+i)^6=8(1-i)(-117+44i)=8(-73+161i)$

Bundan x=584 va y=1288 kelib chiqadi. Demak, $\mathbf{2=0{,}584^2+1{,}288^2}$

47. $h(x)=\frac{ax+b}{-bx+a}$ va $k(x)=\frac{cx+d}{-dx+c}$ kasr-chiziqli funksiyalarning murakkab funksiyasini topamiz: $h(k(x))=\frac{(ac-bd)x+(ad+bc)}{-(ad+bc)x+(ac-bd)}$, va uning koeffitsiyentlari $a+bi$ va $c+di$ kompleks sonlarning ko'paytmasi kabi topiladi.
Berilgan funksiya $f(x)=\frac{x\frac{\sqrt{3}}{2}-\frac{1}{2}}{\frac{1}{2}x+\frac{\sqrt{3}}{2}}$, $z=\frac{\sqrt{3}}{2}-\frac{1}{2}i=\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)$. Shuning uchun g(x) funksiya $z^{2007}=\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)$ shuning uchun $g(x)=-\frac{1}{x}$.

48. $2(z^2+t^2)\geq(z+t)^2$ tengsizlikdan foydalanib topamiz:
$2(1+x+1+y)\geq(\sqrt{1+x}+\sqrt{1+y})^2=4+4a$, bundan isbotlanishi kerak bo'lgan tengsizlik kelib chiqadi.

49. Tenglikdan $a_{n+3}a_n=a_{n+1}a_{n+2}+5$, $a_{n+4}a_{n+1}=a_{n+2}a_{n+3}+5$ topamiz:

$$\frac{a_{n+4}+a_{n+2}}{a_{n+3}}=\frac{a_{n+2}+a_n}{a_{n+1}}$$

shuning uchun juft n da: $\frac{a_{n+2}+a_n}{a_{n+1}}=\frac{a_4+a_2}{a_3}=4$; toq n da: $\frac{a_{n+2}+a_n}{a_{n+1}}=\frac{a_3+a_1}{a_2}=3$
$a_{2n+2}=4a_{2n+1}-a_{2n}$; $a_{2n+3}=3a_{2n+2}-a_{2n+1}$. $\{a_n\}$ ketma-ketlikning barcha hadlari butun.

50. Ixtiyoriy uchburchakda $S=\frac{1}{2}ah_a=\frac{1}{2}bc\sin\alpha$, $a=\sqrt{b^2+c^2-2bc\cos\alpha}$.
Bundan, $h_a=\frac{bc\sin\alpha}{a}=\frac{bc\sin\alpha}{\sqrt{b^2+c^2-2bc\cos\alpha}}$. Endi $b^2+c^2\geq 2bc$ dan foydalansak,
$\sqrt{b^2+c^2-2bc\cos\alpha}\geq\sqrt{2bc(1-\cos\alpha)}=\sqrt{2bc\cdot 2\sin^2\frac{\alpha}{2}}=2\sqrt{bc}\sin\frac{\alpha}{2}$ bundan kelib chiqadiki,
$h_a\leq\frac{bc\sin\alpha}{2\sqrt{bc}\sin\frac{\alpha}{2}}=\sqrt{bc}\cos\frac{\alpha}{2}$, tenglik esa $b=c$ da bajariladi.

51. O'tkir burchakli uchburchak uchun $tg\alpha+tg\beta+tg\gamma=tg\alpha tg\beta tg\gamma$ o'rinli. O'rta arifmetik va o'rta geometric munosabatga ko'ra,
$\frac{tg^5\alpha+tg^5\beta+tg^5\gamma}{tg\alpha\cdot tg\beta\cdot tg\gamma}\geq\frac{3\sqrt[3]{(tg\alpha\cdot tg\beta\cdot tg\gamma)^5}}{tg\alpha\cdot tg\beta\cdot tg\gamma}=3\sqrt[3]{(tg\alpha\cdot tg\beta\cdot tg\gamma)^2}$, ammo
$tg\alpha+tg\beta+tg\gamma\geq3\sqrt[3]{tg\alpha\cdot tg\beta\cdot tg\gamma}$ shuning uchun $tg\alpha tg\beta tg\gamma\geq3\sqrt[3]{tg\alpha\cdot tg\beta\cdot tg\gamma}$ yoki
$\sqrt[3]{tg\alpha\cdot tg\beta\cdot tg\gamma}\geq3$, so'nggi tengsizlikdan foydalanib topamiz: $\frac{tg^5\alpha+tg^5\beta+tg^5\gamma}{tg\alpha+tg\beta+tg\gamma}\geq9$

52. $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ mos ravishda piyoda, velosipedda va mototsiklda yurish tezligi (km/min) bo'lsin. Shartga ko'ra sistema tuzamiz:

29. Agar a va b lar toq son bo'lsa, $c=a^{a+1}+b^{b+1}$ juft va 2 dan katta bo'ladi;
Agar $b=2$ bo'lsa, $c=a^{a+1}+8$ va $a=3k-1$ da c ikki son kublari yig'indisi bo'ladi va u tub bo'lmaydi; $a=3k+1$ da c soni 3 ga bo'linadi; $a=3$ da tekshiramiz, $c=89$-tub son, demak faqat $a=2$ va $b=3$ da; $a=3$ va $b=2$ da.

30. Belgilash kiritamiz, $y=\sqrt[3]{2-x}$ va $z=\sqrt{x-1}$, bundan quyidagi tenglamalar sistemasini topamiz:
$y^3+x=2$, $z^2+1=x$, $y+z=1$. Bundan $z=1-y$ ni topamiz va 1-tenglamaga qo'yamiz, $y^3+y^2-2y=0$, y Є {0,1,-2}
Tenglama uchta 1,2,10 ildizlarga ega.

31. Agar x va y tenglama shartini qanoatlantirsa, 1-tenglama Koshi tengsizligiga teng kuchli,
$xy\geq2\sqrt{\frac{xy}{\sqrt{xy}}}=2\sqrt[4]{xy}$, bundan $xy\geq\sqrt[3]{16}$. Tekshirishlar shuni ko'rsatadiki,
ikkinchi tenglamadan $8(xy)^{\frac{\alpha-3}{2}}\geq2\sqrt{(xy)^{\alpha}}$ bundan, $(xy)^{\frac{3}{2}}\leq4$ yoki $xy\leq\sqrt[3]{16}$.
Shuning uchun $xy=\sqrt[3]{16}$, Koshi tengsizligiga ko'ra $x=y=\sqrt[3]{4}$ bo'ladi.

32. Qiyinchiliksiz isbotlash mumkinki, $a>0$, $b>0$ da $a^3+b^3\geq ab(a+b)$, shunday qilib tengsizlikning chap qismidagi birinchi ifoda $\frac{1}{ab(a+b+c)}$ dan kichik yoki teng, shunday qilib tengsizlikning butun chap qismi quyidagi yig'indidan kichik:
$\frac{1}{a+b+c}\left(\frac{1}{ab}+\frac{1}{bc}+\frac{1}{ac}\right)=\frac{1}{abc}$ tengsizlik isbotlandi.

33. $(x^2+y^2)(z^2+t^2)=(xz+yt)^2+(xt-yz)^2$, berilgan tenglamani quyidagicha yozishimiz mumkin:
$(xt-yz)^2=3(xz+yt)^2$ ammo bu tenglama natural sonlarda yechimga ega emas, chunki 3 butun sonning kvadrati emas.

34. Belgilash kiritamiz, $y=\sqrt{x+\frac{1}{4}}$ va topamiz: $x=y^2-\frac{1}{4}$, $y\geq0$, keyin esa
$y^2-\frac{1}{4}+\sqrt{y^2+\frac{1}{4}+y}=a$,→ $y^2+y+\frac{1}{4}$, → $(y+\frac{1}{2})^2=a$. Bunda $y\geq0$ bo'lgani uchun $a<1/4$ da yechim yo'q; $a\geq1/4$ da esa $y=\sqrt{a}-\frac{1}{2}$.
Tenglama $a<1/4$ da yechimga ega emas, $a\geq1/4$ da esa $x=a-\sqrt{a}$

35. Agar birinchi ifodadagi so'nggi 6 ni 9 bilan almashtirsak, tekshirishlar shuni ko'rsatadiki, ifoda 3 ga teng; ikkinchi ifodadagi so'nggi 6 ni 8 bilan almashtirsak u 2 ga teng bo'ladi, demak yig'indi 5 dan kichik ekan. Boshqa tomondan berilgan ifoda $\sqrt{6}+\sqrt[3]{6}$ dan katta, va tekshiramiz, $\sqrt{6}>2{,}4$ $\sqrt[3]{6}>1{,}6$ shuning uchun ifoda 2,4+1,6=4 dan katta. Demak ifodaning butun qismi 4 ga teng ekan.

36. Faraz qilaylik, n=2k+3 bo'lsin, $2^{2k+3}+2^{2k}=2^{2k}\cdot 9=(3\cdot 2^k)^2$ J: cheksiz ko'p.

37. Shunga teng kuchli tengsizlikni isbotlaylik,

$$\sqrt[n]{n!}>\sqrt[n+1]{(n+1)!}$$
$$(n!)^{n+1}>((n+1)!)^n$$
$$(n!)^{n+1}>(n!)^n(n+1)^n$$
$$n!>(n+1)^n$$

Ammo so'nggi tengsizlik noto'g'ri ekanligi ko'rinib turibdi, demak berilgan tengsizlik noto'g'ri.

38. Osongina topish mumkinki 0, $\frac{1}{2}$, 1 sonlari tenglamaning yechimidir. Agar uning to'rtinchi ildizi ham bo'lsa, Roll teoremasiga ko'ra, $y=(4^x+2)(2-x)-6$ funksiya xosilasi 3 ta nuqtada, ikkinchi xosilasi esa 2 ta nuqtada nolga aylanadi. Ammo, $y'=-(4^x+2)+(2-x)4^x\ln 4$,
$y''=-4^x\ln 4-4^x\ln 4+(2-x)4^x\ln^2 4=4^x((2-x)\ln^2 4-2\ln 4)$ bitta kritik nuqtaga ega.

39. Belgilash kiritamiz, $\sin^2\alpha=a$ va $\cos^2\alpha=b$ va tengsizlikni quyidagicha yozamiz: $(a^k+b^k)(a+b)\le 2(a^{k+1}+b^{k+1})$ va soddalashtiramiz,
$a^kb+ab^k\le a^{k+1}+b^{k+1}$, yoki, $(a^k-b^k)(a-b)\ge 0$. So'nggi tengsizlik to'g'riligi ko'rinib turibdi, tenglik esa $a=b$ da bajariladi.

40. $x\le y\le z$ deylik; agar $x\ge 3$ bo'lganda, $xy+yz+xz\le 3yz\le xyz$, bundan $x\le 2$ kelib chiqadi.
Agar x=1 bo'lsa, tengsizlik $yz<yz+y+z$ ko'rinishda bo'ladi va ixtiyoriy y va zda bajariladi;
Agar x=2 bo'lsa, $\frac{1}{y}+\frac{1}{z}>\frac{1}{2}$ ko'rinishda bo'ladi. $\frac{2}{y}\ge\frac{1}{y}+\frac{1}{z}$, $y<4$, ya'ni y=2 yoki 3 ga teng.
y=2 da $z\ge 2$ da tengsizlik bajariladi; y=3 da $\frac{1}{z}>\frac{1}{6}$ z=3,4,5 bo'ladi.
Javob: $x\le y\le z$ desak $(1,a,b)$ bu yerda a va b- ixtiyoriy sonlar; $(2,2,a)$ $a\ge 2$; (2,3,3) , (2,3,4) , (2,3,5).

41. x ning oldidagi koeffitsiyentni *a* bilan va ozod hadni *b* bilan belgilaylik:

$$a=\frac{1}{100}\left(1-\frac{1}{101}+\frac{1}{2}-\frac{1}{102}+\ldots+\frac{1}{10}-\frac{1}{110}\right)=\frac{1}{100}\left(\left(1+\frac{1}{2}+\ldots+\frac{1}{10}\right)-\left(\frac{1}{101}+\frac{1}{102}+\ldots+\frac{1}{110}\right)\right);$$

$$b=\frac{1}{10}\left(1-\frac{1}{11}+\frac{1}{2}-\frac{1}{12}+\ldots+\frac{1}{100}-\frac{1}{110}\right)=\frac{1}{10}\left(\left(1+\frac{1}{2}+\frac{1}{3}+\ldots+\frac{1}{100}\right)-\left(\frac{1}{11}+\frac{1}{12}+\ldots+\frac{1}{110}\right)\right)=$$

$$=\frac{1}{10}\left(\left(1+\frac{1}{2}+\ldots+\frac{1}{10}\right)-\left(\frac{1}{101}+\ldots+\frac{1}{110}\right)\right)=10a$$

Bundan x=10 kelib chiqadi. J: x=10

42. Belgilash kiritamiz, $y=x^2+7x+a$, tenglamani quyidagicha yozamiz:
$\frac{x}{y}=\frac{y+x}{y-x}$, bundan $xy-x^2=y^2-xy$, x=y kelib chiqadi. Ammo bu mumkin emas, x-y=0 bo'lib qoladi, tenglama yechimga ega emas.

43. Ko'rinib turibdiki x va y musbat. Belgilash kiritamiz, $x=t^2$ va bundan $y=t^3$ kelib chiqadi. Ikkinchi tenglama $t^3+t^2+t=919$ yoki $(t-9)(t^2+10t+91)=0$ ko'rinishga keladi. Bundan, t=9 va x=81, y=729 kelib chiqadi. J: (81,729)

44. Bolalar $x_1<x_2<\ldots<x_9$ ta qo'ziqorin tergan va $x_1+\ldots+x_5>110$ bo'lsin. Bu yerda $x_5\ge 25$: agar $x_5<25$ yoki $x_5\le 25$ bo'lsa, $x_4\le 23$, $x_3\le 22$, $x_2\le 21$, $x_1\le 20$, bundan esa $x_1+\ldots+x_5\le 110$. Shuning uchun $x_6\ge 26$, $x_7\ge 27$, $x_8\ge 28$, $x_9\ge 29$, $x_6+x_7+x_8+x_9\ge 110$, va $x_1+x_2+\ldots+x_9>220$ bo'ladi.
Demak birinchi 5 ta bola 110 tadan kam qo'ziqorin terishgan.

45. Tenglamaning har ikkala tomoniga sinx ni ko'paytiramiz va topamiz:
2cos4xsin2x+sin2x=sinx
sin6x-sin2x+sin2x=sinx
sin6x=sinx, $x_1=\frac{2\pi n}{5}$ va $x_2=\frac{2k+1}{7}\pi$ (n,k Є Z) .

Biz sinx ga ko'paytirganda x=πm chet ildizni hosil qildik, shuning uchun n va 2k+1 sonlari 5 ga va 7 ga bo'linmasligi kerak,
demak, $n\neq 5m$, $k\neq 7d+3$

46. $m_a^2+m_b^2+m_c^2=\frac{3}{4}(a^2+b^2+c^2)$ Berilgan tengsizlik quyidagiga teng kuchli:
$\frac{3}{4}(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)\ge 27S^2$ yoki, $(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)\ge 36S^2$
Boshqa tomondan $a\le b\le c$ bo'lsin, $h_a\ge h_b\ge h_c$ bo'ladi. Chebishev tengsizligidan topamiz:
$(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)\ge 3(a^2h_a^2+b^2h_b^2+c^2h_c^2)=36S^2$
Tenglik sharti esa $a=b=c$ da bajariladi.